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Entry Regulations and Product Variety in Retail

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Abstract

This paper estimates a dynamic model of store adjustments in product variety that considers multiproduct service technology to evaluate the impact of entry regulations on variety and long-run profits in Swedish retail. Using rich data on stores and product categories, we find that more liberal entry regulation increases productivity and decreases the adjustment costs of variety. Counterfactual simulations of modest liberalizations of entry incentivize incumbents to offer more product categories to consumers while increasing efficiency and long-run profits. Regional differences are reduced as consumers and incumbents obtain more benefits in markets with restrictive regulation. Generous liberalizations of entry induce net exit of product categories and harm incumbents in markets with limited demand.

Keywords: Retail markets; entry regulation; product variety; productivity; competition.
JEL Classification: L11, L13, L81.

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1 Introduction

An important goal of policymakers is to ensure that consumers enjoy broad access to products and services regardless of where they live. To this end, the appropriate design of entry regulations has been widely debated among policymakers and academics.[1] The choice of product variety is endogenous, where firms trade off short-run costs and long-run benefits.[2] The extent to which there is too much or too little product variety as a result of entry regulations is theoretically ambiguous and can only be assessed by empirical work. Yet, there is remarkably little research on the incentives for product repositioning and adjustment in inputs after regulatory changes, particularly for service industries characterized by economies of scale and scope.

In this paper, we estimate a dynamic model of store adjustments in product variety and inputs to evaluate the impact of entry regulations on variety and long-run profits. The model builds on a multiproduct service technology, endogenizes stores' product variety decisions, and quantifies the long-run store benefits of expanding variety. The model is estimated using rich Swedish retail data on product categories, stores and entry regulations across local markets for the period 2003-2009. Then, we use counterfactual analysis to examine the dynamic response to alternative regulatory regimes that encourage product variety in markets with restrictive regulation or in rural locations.[3] This conveys knowledge for designing policy tools to improve variety and employment and to equate living conditions across regions, being highly prioritized among policymakers.

Entry regulations and government subsidies are common in OECD countries, but the design and stringency differ across countries.[4] According to the Swedish Plan and Building Act (PBL), all stores are subject to the regulation, and each municipality has the power to make land-use decisions. Local authorities typically require each store to complete a formal application when seeking entry. The application is approved or rejected after the potential consequences of entry on factors such as market shares and product variety have been evaluated. Rarely are all applications approved in Sweden. We follow the previous literature and use the number of approved PBL applications divided by population density to measure regulatory stringency and provide solutions

[1]See, for example, European Competition Network (2011), European Commission (2012) and the survey of regulations in retail conducted by Pozzi and Schivardi (2016).

[2]We measure product variety as the number of product categories when there is no data on all products in a category (i.e., the range of products in a category). The marketing literature often uses rich product data and refers to variety as a product mix consisting of multiple product lines (categories). The number of product lines refers to product width, whereas the number of products in a product line refers to product depth (range). The sum of product depth across all firm's product lines defines product length, which can be used to measure product variety when data allows.

[3]Rural and urban markets are defined by population. Markets with restrictive and liberal entry regulation are defined by the stringency of regulation.

[4]Countries like the United States have more flexible zoning laws, while the United Kingdom and France explicitly regulate large entrants.

to endogeneity concerns.

Our model captures a new mechanism behind the dynamic effects of regulations, recognizing retailers' incentives for repositioning inputs and innovating to offer more products to sell. The novelty of our dynamic framework is that we model economies of scope and store input allocation and consider the adjustment costs related to offering product variety. Fiercer competition caused by more liberal regulations increases firms' incentives to run efficient operations by investing in new technology and adjusting their inputs.[5] We allow entry regulations to influence future productivity and adjustment costs related to offering product variety.[6] Higher productivity and lower adjustment costs related to offering variety create incentives for stores to introduce new products. Economies of scope make it cheaper to sell many product categories together than selling them separately and can arise from cross-selling products using the same employees and systems (machinery and equipment) or business sharing centralized functions such as finance and marketing. New products are introduced if the expected long-run gains are higher than the cost of adding variety, implying that new products do not merely cannibalize sales from existing products. The magnitude of the induced changes from more liberal entry regulation on the number of product categories (extensive margin), sales per category (intensive margin), productivity and long-run store profits can only be determined through empirical work.

Product-level data connected to a census are rarely available for service industries. We access such data and use product categories to measure product variety at the store level. Facts in our data guide the formal model. Stores frequently adjust their products. There is substantial variation in product categories and simple performance measures such as labor productivity within and between stores over time. Reduced-form regressions show that more liberal regulation is associated with more product categories and improved store performance measures. More liberal regulation is also associated with a larger sales increase among a store's bottom-selling categories than among top-selling categories.

In the dynamic model, stores choose product categories, labor, inventory and investment in technology based on store-specific supply and demand primitives and characteristics of the local market (e.g., Jovanovic, 1982; Hopenhayn, 1992; Ericson and Pakes, 1995). First, we recover store revenue productivity and demand shocks affecting product-category sales and market shares using multiproduct technology and a control function estimator at the product-category level relying on input demand for labor and

[5] See, e.g., Joskow and Rose (1989), Bertrand and Kramarz (2002), Suzuki (2013), Turner et al. (2014), Pozzi and Schivardi (2016), Maican and Orth (2018).

[6] The empirical literature often find positive effects of stronger competition on productivity due to external factors such as trade liberalization and less restrictive regulation, e.g., De Loecker (2011), Syverson (2011), Maican and Orth (2015), Maican and Orth (2017), and Backus (2020).

inventory.[7] Demand shocks can be associated with consumers' quality of the shopping experience and other demand factors that affect store sales and market share, and their evolution is not under the store's control. We discuss identification in detail and provide Monte Carlo simulations. Second, we solve the store's dynamic optimization problem and identify the adjustment costs of product categories by matching the observed data with the prediction of the model. Third, counterfactual simulations solve for the optimal number of product categories and sales per category and quantify the long-run benefit of variety under alternative regulations and government subsidies.

This study is, to the best of our knowledge, the first empirical research that analyzes how regulations affect stores' incentives to offer product variety using a single-agent dynamic framework where economies of scope are embedded in multiproduct technology. Early work by Baumol et al. (1982) models the cost side to understand variety. While early work on product differentiation imposed restrictive assumptions, the demand literature employs rich modeling of consumer behavior to understand market performance with multiproduct firms.[8] Prices restrict demand in terms of quantity, while purchasing costs related to traveling and waiting in checkout lines limit demand in terms of product variety (Bronnenberg, 2015). Stores reduce purchasing costs and provide more convenience by increasing shopping quality, which increases fixed costs and mitigates variety. Entry papers consider that firms pay a fixed cost to increase variety, but this does not fully explain why service firms offer multiple products (Bailey and Friedlaend, 1982). In this paper, we demonstrate the incentives for product repositioning after regulatory changes using a dynamic framework of multiproduct technology and considering demand shocks in a local market environment.

This paper differs from previous literature in that our framework endogenizes store decisions over product variety by integrating multiproduct technology into a fully dynamic model with adjustment costs related to offering product variety. The proposed multiproduct technology used by stores to generate sales is transparent over the aggregation across products and the rate of substitution between products and is consistent with stores' profit maximization behavior, as discussed in the early theoretical literature on production technology (Hicks, 1946; Mundlak, 1964; Fuss and McFadden, 1978). We provide an empirical tractable model that adds to previous research on entry regulations

[7]See Olley and Pakes (1996), Doraszelski and Jaumandreu (2013), Maican and Orth (2017), and Kumar and Zhang (2019). Kumar and Zhang (2019) use the cost of goods to recover the distribution of demand shocks in manufacturing, but do not model the firm's product variety or recover a demand shock for each firm.

[8]Early models of product differentiation assumed that firms produce a single product and variety reduced to the number of firms (Spence, 1976; Dixit and Stiglitz, 1977; Mankiw and Whinston, 1986). The demand literature uses rich modeling of consumer behavior, but it does not model how firms use their internal resources to offer variety. For example, Anderson and De Palma (1992), Verboven (1996) and Anderson and De Palma (2006) present theoretical nested demand frameworks where consumers first decide on the firm, then which product and how much to buy.

and firm performance using strategic interactions where the computational burden limits the degree of product differentiation (e.g., Suzuki, 2013; Fan and Xiao, 2015; Maican and Orth, 2018). The paper also links to the scarce literature on variety responses to regulatory changes that stress the demand side and to recent work that uses dynamic structural models to examine the firm's response to industry policies (e.g., Ryan, 2012; Sweeting, 2013; Fowlie et al., 2016; Barwick et al., 2018). Stores in our model respond differently to changes in regulations and a store's market share is determined by its own product variety and that of rivals in local markets. In this regard, the paper also connects to the literature on competition and variety that typically finds a positive relationship but without considering the allocation of inputs explicitly.[9]

We also contribute to the literature on productivity and multiproduct technology, which often relies on exogenous product variety and ignores the dynamic aspects of adjusting variety. In particular, we contribute to recent work on multiproduct firms and productivity in manufacturing using data on sales and physical quantities (e.g., De Loecker et al., 2016; Dhyne et al., 2017). Our model adapts several features typical for services that should affect the response to regulatory changes. Retailers frequently change product variety using the same technology and utilizing economies of scale and scope. The nature of services makes it difficult to measure physical quantities and prices, and to aggregate across products, complicating the definition of technical productivity (Oi, 1992). We add to the understanding of revenue productivity dynamics in services by recovering two store-level unobservables and their relationship as in Maican and Orth (2021), which do not model endogeneity of store variety, adjustment costs of variety and the effect of regulation.

The results of the structural model show that entry regulations are a key determinant of stores' optimal product variety. The median adjustment cost of product categories is 29 percent higher in markets with restrictive rather than liberal regulation. Stores located in restrictive markets have the highest dispersion in the long-run benefit of adding one more product category. The median benefit is approximately 1 percent lower in restrictive than in liberal markets. The median benefit of adding variety is 2 percent lower for stores located in rural rather than urban markets, reflecting less variety to consumers in rural areas.

Counterfactual policy experiments show that more liberal entry regulation forces incumbents to reallocate inputs and reposition product variety, which increases product-category entry rates. Modest liberalization of entry regulation increases incumbents' long-run profits due to productivity advances, lower adjustment costs and modified product categories. The improvements among incumbents as well as product-category

[9] See also Ellickson (2007), Watson (2009), Ren et al. (2011), Basker et al. (2012), Bronnenberg and Ellickson (2015), Hortacsu and Syverson (2015), Berry et al. (2019), Hsieh and Rossi-Hansberg (2019), and Maican and Orth (2021).

benefits for consumers are greatest in markets with restrictive regulation. Consequently, such a regulatory regime adequately reduces regional differences. Simulations of doubling the number of accepted PBL applications show high product-category entry rates but even higher product-category exit rates in markets with limited demand. Although incumbents are incentivized to improve their operations, it cannot outweigh the loss in sales from intense competitive pressure, implying that long-run profits decrease. Counterfactuals show that a cost subsidy to stores utilizing economies of scope can ensure product variety in rural markets but that the governmental cost can be high.

Section 2 presents the entry regulations, data and reduced-form evidence. Section 3 presents the dynamic model and empirical framework. Section 4 discusses the empirical results and Section 5 the counterfactual experiments. Section 6 summarizes the paper. In several places we refer to an online Appendix.

2 Swedish retail trade and entry regulations

The goal of policymakers is to ensure that all individuals in society have access to a wide variety of products at low prices and in stores within a reasonable geographic distance. To reach this goal, most OECD countries empower local governments to make decisions regarding the entry of new stores. The Swedish Plan and Building Act [PBL] regulates the use of land, water and buildings. The regulation contains a comprehensive plan that covers and guides the use of the entire municipality and detailed development plans that cover only a fraction of the municipality. The detailed development plans divide municipalities into smaller areas for which limits on use and design are set, i.e., construction rights for real estate and whether areas can be used for workplaces, housing, schools, parks, etc. Entering a new store requires that the PBL admits operations of retail activities in the geographic area where the store wants to enter. A formal application needs to be sent to the municipal government that is supposed to evaluate consequences on prices, accessibility of store types and products for different consumer groups, traffic, broader environmental issues, etc. The local government can accept or reject an application. Because the Swedish regulation is typical for many other countries, our application to Sweden is relevant and offers broad implications for other countries (Pozzi and Schivardi, 2016). Appendix D provides an extensive discussion on PBL in Sweden.

Regional development policies. Regional subsidies are alternative policy tools employed to encourage stores to provide a wide variety of products. In 2001, the Swedish government announced a new regional development policy designed to maintain a sustainable service level in all parts of Sweden (the bill 2001/02:4 *A policy for growth and* *viability for the whole country).* One of the programs embedded in the policy was *Stores in the countryside.* The aim of the program was to improve stores in rural areas by implementing store performance actions, such as store refitting, improving the distribution of products and technical equipment, and modernizing inventory, and assigning mentors to enhance communication between store managers and local authorities. In 2015, the Swedish government announced *The Rural Development Programme* [RDP]. The RDP contains support and compensation for municipalities to achieve objectives, such as a balanced territorial development of rural economies and communities as well as improved quality of life. The RDP aims to make it easier to live and operate businesses in rural areas by investing in local services and technologies (e.g., broadband). The RDP emphasizes the importance of retail stores, as they also provide numerous other utilities, such as postal services. The stringency of entry regulations is crucial for achieving the goals of RDP because investments in infrastructure and access to services are involved in entry regulations in great detail.

Local markets. Sweden consists of 290 municipalities that make decisions regarding entry regulations and regional development policies. Following previous studies on Swedish retail and considering the fact that municipal governments decide over entry and regional development programs, here, a municipality refers to a local market (Maican and Orth, 2015, Maican and Orth, 2018).[10] We classify municipality in market types. The first classification rests on the stringency of entry regulations. Markets with regulatory stringency below the median value are defined as restrictive; otherwise they are defined as liberal. The second classification is rural or urban markets. Markets with less than 10,000 inhabitants are defined as rural; otherwise they are defined as urban. The restrictive and liberal markets are defined on the potential competitive pressure from entrants, whereas the rural and urban markets are defined based on potential demand capabilities. Because entry regulations affect all types of markets and regional programs target rural markets, our market types are crucial for understanding the development of local markets under numerous policy changes.

Data. The empirical application focuses on the three-digit industry, *Retail sale of new goods in specialized stores* (Swedish National Industry (SNI) code 524). This retail sector includes the following subsectors at the five-digit SNI: clothing; furniture and lighting equipment; electrical household appliances and radio and television goods; hardware, paints and glass; books, newspapers and stationery; and other specialized stores.

We use three data sets provided by Statistics Sweden and the Swedish Mapping, Cadastral and Land Registration Authority (SMA). The first data set covers detailed annual information on all retail firms in Sweden (census) during the period 2000 to 2010.

[10]A municipality consists of one or more localities. Statistics Sweden [SCB] also defines trade areas for retailers based of the number and the size of retailers, i.e., to have at least five retail trade stores or four retail trade stores that have together having at least 100 employees (SCB, 2015).

The data contain financial statistics of input and output measures: sales, value-added, the number of employees, capital stock, inventories, cost of products bought, investment, etc. Inventories capture the value of products held in stock in the end of each year and are taken from book values (accounting data). The cost of products bought measures store's cost of buying products from the wholesaler. The cost of products bought and inventories both rely on the input prices of goods, i.e., they are based on what stores pay to the wholesaler. In other words, sales and value-added are measured in output prices, whereas the cost of products bought and inventories are measured in input prices. Because of difficulties in measuring quantity units in retailing arising from the nature and complexity of the product assortments, quantity measures of output and inventories are not available.

Our second data set includes information on approximately 1,100 stores per year and covers store-level data on all product categories and their yearly sales from 2003 to 2009. Unique identification codes allow us to perfectly match the product categories to the stores. The product categories have 6-8 digit codes assigned, which define categories such as clothes for women, clothes for men, and clothes for children.[11] The number of product categories is our measure of product variety in a store. That is, the number of product categories captures the extensive margin of product variety in a store. Data on sales per product category capture the intensive margin of product lines (range) inside a category. Most importantly, the combination of the two data sets allows us to compute product market shares inside a store and a store's market share in a geographic market (municipality), which provides rich information on the local market structure.

The third data set contains data on the number of applications approved by local authorities for each municipality and year (SMA). This data set also includes applications to alter land-use plans and the total number of existing land-use plans. We follow previous literature on land use and entry regulations and define the stringency of regulation in local markets as the number of approved PBL applications divided by the population density (Bertrand and Kramarz, 2002; Suzuki, 2013; Turner et al., 2014; Pozzi and Schivardi, 2016; Maican and Orth, 2018).[12]

Descriptive statistics and stylized facts. Table 1 shows that there is an aggregate increase in sales, value-added, the average number of product categories, investments, and labor over time. From 2005 to 2009, sales increased by 36 percent, investments by 53 percent and the number of employees by 21 percent. An average store has approximately 4 product categories. The number of product categories varies between 1

[11]The product data set follows a similar classification system to the one used for the sample data collected on prices and quantities in manufacturing. The complexity of measuring physical quantities and aggregating across products makes it difficult to define an annual price index for a product category.

[12]Municipalities with a nonsocialist majority approve more PBL applications. The correlation between nonsocialist seats and the number of approved PBL applications in local markets is 0.6.

and 17 in our sample. Our regulation measure, i.e., the average number of approved PBL-applications over population density, increased from 0.23 to 0.29 during our study period. That is, an increase from 23 to 29 approved applications per 100 square kilometers. That more approvals are associated with fiercer competition is confirmed by the negative correlations over time between sales per product category and our regulation measure.

Retailers often adjust their product categories to improve the store's competitiveness and adapt to the local market environment. Product repositioning is more frequent in retail than in manufacturing because retailers employ the same technology to sell a different set of product categories. In our sample, we observe adjustments in product categories in 52 percent of store-year observations, a result also confirmed by the median number of years a store adjusts product categories that is approximately half of the total number of years in the sample. Nevertheless, the mean of cumulated yearly adjustments of the number of product categories is positive (i.e., product variety increases over time). Yearly adjustments in the store's number of product categories between $t-1$ and t vary considerably. The interquartile range of yearly changes in the store's number of product categories is 2. We also find substantial variation in the yearly changes in the number of product categories across five-digit subsectors, i.e., the median of the five-digit interquartile range is 1, and the maximum is 3.

Figure 1 presents box-plot charts showing the distributions of store performance measures before and after the acceptance of new PBL applications. We measure store performance by labor productivity (log of sales per employee), market share, and inventory performance (the log of sales per average inventory and the log of cost of goods sold over average inventory). Median labor and inventory productivity is higher, whereas median market share is lower after acceptance of new PBL applications. This suggests a positive relationship between increasing competition from more liberal regulation and store performance, in line with previous literature. It also suggests that we have to control for entry regulation when developing more sophisticated measures of store performance such as total factor productivity.

The box plots in Figure 2 show that the median store has more product categories and higher sales per product category after the acceptance of new PBL applications. Consumers benefit from more product variety, and incumbents benefit from higher sales per product category in markets with more liberal regulation. However, the drivers of these patterns are unclear without a modeling framework.

Reduced-form regressions show that acceptance of new PBL applications is associated with changes in the number of product categories and the distribution of sales across product categories in a store. To measure store diversification in terms of sales, we compute the entropy measure for each store j based on the market share of each product category i sold by the store, $E_{jt} = \sum_i ms_{ijt} ln(ms_{ijt})$ (Bernard et al., 2011). A store that focuses on top sales categories has a large entropy. Table 2 shows that new PBL applications increase the number of product categories and decrease the entropy of product sales. On average, stores in markets with new applications accepted have approximately 5 percent more product categories and 7 percent lower product-sales entropy. This is suggestive evidence that regulatory changes are associated with adjustments in product variety.

To investigate the dynamic effects of entry regulations on the number of product categories and sales entropy, we use $AR(1)$ reduced-form regressions that include year, subsector, and local market fixed effects (i.e. $\Delta z_{jmt} = \alpha_z z_{jmt-1} + \alpha_r r_{mt-1} + f_s + f_t + f_m + u_{jmt}$).[13] Table 3 shows that one additional PBL per population density increases the number of product categories in stores by 4.7 percent and decreases stores' product-sales entropy by 5.2 percent. The average persistence in the number of product categories and sales entropy are approximately 60 and 63 percent, respectively.

Our results are robust to considering the endogeneity of the entry regulation measure. Specifications (3), (6) and (9) in Table 3 control for the possible endogeneity of entry regulation using an instrumental variable approach. We use three instruments based on previous literature: the share of nonsocialist seats in the local government (Maican and Orth, 2015, Pozzi and Schivardi, 2016), the number of approved applications in the neighboring municipalities, and one internal instrument based on exogenous variables to stores (e.g., income and income squared) (see Lewbel, 2012 for a discussion on internal instruments). The first instrument relies on nonsocialist local governments being more positive for entry. To be an effective instrument for entry regulation, the share of nonsocialist seats should not be related to local market demand. This instrument raises the following concerns. First, the outcomes of elections might be influenced by economic conditions. Political business cycles can only affect our results if there is a substantial ability to predict future demand shocks when politicians are elected. The second concern is that political preferences might capture local policies other than entry regulations. In Sweden, PBL is rather exceptional because it enables local politicians to play a key role. Furthermore, in our context, the number of PBLs in other markets is an appropriate instrument if it reflects common trends or demand shocks that are specific only to entry regulations. Although the proposed instruments are not perfect, we believe that they are the best instruments, given previous work and the available data.

The results of the Sargan test shows that the overidentifying restrictions are valid, i.e., the test fails to reject the null hypothesis that the instruments are uncorrelated

[13] z is one of the following variables: the number of product categories, the logarithm of the number of product categories, and sales entropy.

with the remaining shocks. We also report the partial F-test, as suggested by Staiger and Stock (1997). The statistically significant F-tests show that the instruments are not weakly correlated with the entry regulation measure.

3 A model of multiproduct service technology and entry regulations in retail

We consider a retail sector where all stores focus on a well-defined service activity (e.g., selling apparel or selling shoes). Based on the observed information at the beginning of period t, stores choose product categories, inventory adjustments, labor, and investments in technology to generate sales. First, we introduce a multiproduct technology and discuss its theoretical foundations. Second, we construct a product-category sales-generating function and recover two store-specific unobservables for the researcher (i.e., revenue productivity and demand shocks). Although we measure product variety by the number of product categories in a store, our model can allow for modeling of individual product-level data linked to a census if available. Third, we model and solve the store's dynamic optimization problem, highlighting the dynamic role of entry regulations and adjustment costs for incumbents' endogenous decisions on product variety.

3.1 Multiproduct service technology in retail

Retailers offer multiple products and services to consumers. The existence of economies of scale and scope is the main determinant of the existence of multiple products at the firm/store level (Panzar and Willig, 1981; Bailey and Friedlaend, 1982). The multiproduct characteristic creates difficulties in aggregating the service output when there is not a single value function because the composite service output of a store depends on other things, including prices. In addition, the productivity of resources in a product or service is not independent of the level of services in other products in retail.

ASSUMPTION 1: *The multiproduct service-generating function of a retailer can be written as an implicit function, which can be described by the transcendental function that generalizes the Cobb-Douglas function (Mundlak, 1963; Mundlak, 1964):*

$$F(\mathbf{Q}, \mathbf{V}) = G(\mathbf{Q}) - H(\mathbf{V}) = 0 \tag{1}$$

where $G(\mathbf{Q}) = Q_1^{\tilde{\alpha}_1} \times \cdots \times Q_{np}^{\tilde{\alpha}_{np}} exp(\tilde{\gamma}_1 Q_1 + \cdots + \tilde{\gamma}_{np} Q_{np})$; $H(\mathbf{V}) = V_1^{\tilde{\beta}_1} \times \cdots \times V_m^{\tilde{\beta}_m} exp(\tilde{\omega})$; $\mathbf{Q}$ is the vector of service output (i.e., product categories in our case); Q_i is the i-th service output of the store (i.e., quantity of product category i), $i = \{1, \cdots, np\}$; V_e is the e-th service input of the store (e.g., labor, capital, inventories), $e = \{1, \cdots, m\}$; and $\tilde{\omega}$ is the retailer's technical productivity (i.e., quantity-based total factor productivity).[14] As we discuss below, parameters $\tilde{\alpha}_1, \cdots, \tilde{\alpha}_{np}$ and $\tilde{\gamma}_1, \cdots, \tilde{\gamma}_{np}$ define the production frontier and affect product-product and product-input substitutions, playing a key role in profit maximization, and $\tilde{\beta}_1, \cdots, \tilde{\beta}_m$ affect product-input and input-input substitutions.

In the following, we use i to index the service outputs (product categories) and e to index the inputs. The assumption regarding the transformation function $G(\mathbf{Q}) - H(\mathbf{V}) = 0$ is known as the separability property, and it has key implications in empirical applications. First, this assumption implies that almost always the retailers sell the product categories jointly. That is, the product categories cannot be sold separately using a sales technology for each product category (nonjoint sales). Second, it can be shown that a necessary and sufficient condition for separability is that the total cost function is multiplicatively separable (in quantity and input prices), which implies that the ratio of two marginal costs is independent of input prices (Hall, 1973).[15] Under competitive equilibrium, this implies that product-category price ratios depend on the product-category mix. Third, a necessary and sufficient condition for nonjointness is that the total cost of selling all product categories is the sum of the cost of selling each product category separately. Therefore, nonjointness in sales technology is restrictive in retail because economies of scale and scope are not modeled explicitly (Panzar and Willig, 1981). Furthermore, it also implies that marginal cost ratios are independent of the product-category mix.

In empirical applications, the theoretical results of the multiproduct service function related to profit maximization play a crucial role in the identification of sales technology. For example, productivity is typically defined as aggregate output over aggregate inputs; that is, the output and input coefficients $\tilde{\alpha}_i$ and $\tilde{\beta}_j$ affect the productivity measure. For simplicity of exposition of the multiproduct technology, we assume that the prices are given and focus on no adjustment cost in inputs. We relax this assumption in the empirical setting, which allows for dynamic inputs such as capital stock and inventories. The static profit maximization problem at the store level is given by

$$\begin{gathered} \max_{\mathbf{V}} \Pi = \mathbf{P}'\mathbf{Q} - \mathbf{W}'\mathbf{V} \\ F(\mathbf{Q}, \mathbf{V}) = 0 \end{gathered} \tag{2}$$

where $\mathbf{P}$ and $\mathbf{W}$ are vectors of output and input prices, respectively.

In the case of two inputs and two outputs, Mundlak (1964) shows the restrictions on the coefficients of transcendental multiproduct functions that are required to satisfy

[14]See Hicks (1946) for an early discussion on the general implicit production function. By introducing the exponential term in $G(\cdot)$, we destroy homogeneity of $H(\cdot)$, but allow for inflexion points in the function (Halter et al., 1957).

[15]Hall (1973) proposes a multiproduct cost function specification where separability and nonjointness are introduced as parametric restrictions.

the static profit maximization conditions. We provide a general result and show that these restrictions are valid when there are more than two outputs and inputs. A reader not interested in theoretical details can move directly to Section 3.2.

THEOREM 1: *Consider a general service-generating function* $F(\mathbf{Q},\mathbf{V}) = G(\mathbf{Q}) - H(\mathbf{V}) = 0$, *where* $G(\mathbf{Q}) = Q_1^{\tilde{\alpha}_1} \times \cdots \times Q_{np}^{\tilde{\alpha}_{np}} exp(\tilde{\gamma}_1 Q_1 + \cdots + \tilde{\gamma}_{np} Q_{np})$; $H(\mathbf{V}) = V_1^{\tilde{\beta}_1} \times \cdots \times V_m^{\tilde{\beta}_m} exp(\tilde{\omega})$. *If the parameters satisfy the following conditions: (a)* $\tilde{\alpha}_i < 0$ *and* $\tilde{\gamma}_i > 0$ *for all* $i = \{1, \cdots, np\}$; *(b)* $\tilde{\beta}_e > 0$ *for all* $e = \{1, \cdots, m\}$, *then the conditions for profit maximization are satisfied.*

PROOF: The main idea of the proof is that the sign of the determinant of the bordered Hessian matrix of the optimization problem (2) should satisfy the second-order requirement for profit maximization. The proof and an additional discussion are provided in the Appendix A for individuals interested in the technical details.■

The introduction of the $\tilde{\gamma}_i$ parameters plays a key role in understanding the properties of the multiproduct function and their empirical implications. For certain values of $\tilde{\gamma}_i$, the service output is sold at the minimum cost and the optimal inputs yield minimum revenues. In the multiproduct case, we want to avoid these situations (saddle points). Proposition 1 describes these cases.

PROPOSITION 1: *If the service function is simple Cobb-Douglas in outputs* ($\tilde{\gamma}_i = 0$ *for all* i) *and inputs and the first-order conditions are satisfied, then the optimal service quantity* $\mathbf{Q}^*$ *is sold at the minimum cost and any inputs* $\mathbf{V}^*$ *yield minimum revenues. The profit* $\pi(Q^*, V^*)$ *at the point* (Q^*, V^*) *is a saddle point, i.e.,* $\pi(Q^*, V) \leq \pi(Q^*, V^*) \leq \pi(Q, V^*)$.

PROOF: The proof uses the sign of the determinant of the Hessian matrix. For the full proof and an additional discussion, we refer readers interested in the technical details to Appendix A (see also Mundlak, 1964).■

A direct consequence of Proposition 1 is that when the inputs $\mathbf{V}$ produce minimum revenues and the first-order conditions are satisfied, then the profit can be maximized by a selection of product categories, i.e., a corner solution. This problem does not exist in the case of a single output (i.e., product category). The condition $\tilde{\alpha}_i < 0$ and $\tilde{\gamma}_i > 0$ for all i is not the only second-order condition for profit maximization.[16] Another key aspect of a multiproduct technology is that the sign of the parameters $\tilde{\gamma}_i$ determines the sign of the product category (factor) substitution (see Appendix A). The marginal rate of substitution for $\tilde{\gamma}_i = 0$ implies that the product-product marginal rate of substitution is a convex function. This function is concave when $\tilde{\gamma}_i > 0$, which has key implications in empirical applications that allow for economies of scope.

Aggregation and the role of sales. To write the service-generating function at the

[16] It is important to note that the result in Theorem 1 holds when some $\tilde{\alpha}_i$ are positive (not all) and, in this case, the corresponding $\tilde{\gamma}_i$ can be set to zero, which can be useful to reduce the number of parameters to be estimated.

product-category level, we need to normalize one parameter to one, say the i-th output, which can be done by raising the service function to the power of $-\tilde{\alpha}_i$. In this case, the resulting parameters of product categories other than i will have a reverse sign when $\tilde{\alpha}_i$ is negative. When the quantity is not observed, we want to set the weights γ_i to obtain a meaningful interpretation of the aggregation across the store's product-category mix. As suggested in Mundlak (1964), we consider $\tilde{\gamma}_i = \tilde{\alpha}_y P_i$, where P_i is the price index of product category i (price of a representative basket), which yields the product-category sales and reduces the number of parameters to be estimated. Thus, $\sum_{i=1}^{np} \tilde{\gamma}_i Q_i = \tilde{\alpha}_y \sum_{i=1}^{np} P_i Q_i = \tilde{\alpha}_y Y$, which is total store-level sales Y multiplied by $\tilde{\alpha}_y$, and it has a meaningful interpretation. The store's total sales thus play a key role in the relationship between inputs and product categories for the multiproduct service-generating function because it drives substitution between product categories. We use this result from the transcendental production functions to write a product-category sales-generating function that accounts for sales of other products.

3.2 Empirical framework: Multiproduct sales-generating function

We start the empirical framework by modeling a multiproduct sales-generating function accounting for local entry regulations. Without loss of generality, we write the model at the product-category level using the simplest demand setting. If one accesses data on product categories and products inside a category, one can derive product-level sales accounting for the nested structure.

ASSUMPTION 2: *All stores use the same service technology to sell their product categories, and this technology does not depend on the product.*

Based on transcendental technology (1), the multiproduct service-generating function of store j in logs is given by

$$\sum_{i=1}^{np_j} \tilde{\alpha}_i q_{ijt} + \tilde{\alpha}_y Y_{jt} = \tilde{\beta}_l l_{jt} + \tilde{\beta}_k k_{jt} + \tilde{\beta}_a a_{jt} + \tilde{\omega}_{jt} + \tilde{u}^p_{jt}, \tag{3}$$

where q_{ijt} is the logarithm of the quantity of product category i sold by store j in period t, Y_{jt} represents the total sales of store j in period t, l_{jt} is the logarithm of the number employees, k_{jt} is the logarithm of capital stock, a_{jt} is the logarithm of the sum between the inventory level in the beginning of period t (n_{jt}) and the products bought during the period t, and $\tilde{u}^p_{jt}$ are the remaining service output shocks.[17] Assumption 2 allows us to reduce the number of parameters to be estimated in empirical applications. With sufficient data for all product categories across markets over a long period of time, assumption 2 can be relaxed to allow separate technologies for each product. Because

[17] We follow the common notation of capital letters for levels and small letters for logs.

each store is unique in our data, we omit the local market index m if the store index j is present and refer to store j in market m.

In a multiproduct setting, the sales technology possibilities requires aggregation over the different products. We need product prices to use product sales to aggregate over products. In many data sets, product-level prices are commonly not observed for all products; therefore, researchers have used the equilibrium price from a demand equation to model sales. A product category consists of physical products and store-specific services associated with each product. Two stores that sell product categories having the same label (e.g., furniture for kitchen) do not sell exactly the same products in our model. Even if stores sell the same product brands in a category, it is unlikely that they offer the same purchase service to consumers for each product. In our model, the total number of product categories across stores in a local market is the choice set of a consumer. For simplicity of exposition, we assume that consumers have constant elasticity of substitution (CES) preferences over differentiated product categories. As in our data and many empirical settings, the researcher observes product information only for a sample of stores and total sales for all stores in local markets. The set of product categories from stores with the same service activity in a local market for which the researcher does not have product information defines the consumer's outside option.

The consumer's decision is how much to purchase of each product category from stores with product information available and from the outside option. The link between a CES demand system and a discrete choice demand system is used to write the consumer choice probability equation consistent with CES preferences[18]

$$q_{ijt} - q_{ot} = \mathbf{x}'_{ijt}\boldsymbol{\beta}_x + \sigma_a a_{jt} - \sigma p_{ijt} + \tilde{\mu}_{ijt}, \tag{4}$$

where p_{ijt} is the logarithm of the price of product category i in store j; $\mathbf{x}_{ijt}$ represents the observed determinants of the intensive and extensive margins of the utility function when the consumers buy the product category i from store j, σ is the elasticity of substitution, $\tilde{\mu}_{ijt}$ represents the unobserved product characteristics at the store level, for example, the quality of the shopping experience attached to product i in store j, and q_{ot} is the outside option quantity.[19] The presence of a_{jt} in a demand equation captures the fact that consumers prefer stores with products in stock.

Multiplying the price p_{ijt} from (4) by the output weights (elasticities) $\tilde{\alpha}_i$, summing up over the number of products, and using the result in (3), we obtain the sales-generating

[18]See, e.g., Anderson et al. (1987), Anderson and De Palma (2006), and Dube et al. (2020). Dube et al. (2020) provide an extensive discussion on the link between CES and discrete choice demand approaches. The demand system is similar to the logit discrete choice system based on unit demand, but the logarithm of price is used. A nested demand framework can be integrated, but the form of the sales-generating function will include more terms.

[19]σ is globally identified for the set of products with positive individual choice probabilities because this system satisfies the connected substitutes condition provided by Berry et al. (2013) and is invertible.

function at the store level that is used to obtain the sales for product i, y_{ijt}

$$y_{ijt} = -\alpha_y y_{-ijt} + \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \beta_q y_{ot} + \mathbf{x}'_{jt}\boldsymbol{\beta}_x + \omega_{jt} + \mu_{jt} + u^p_{ijt}, \qquad (5)$$

where y_{-ijt} is the logarithm of sales of product categories other than i, y_{ot} measures the sales of the outside option, $\mathbf{x}_{jt}$ sums all observed characteristics at the store and market levels, and u^p_{ijt} represents i.i.d. remaining shocks to sales that are mean-independent of all control variables and store inputs. We show the derivation of equation (5) in Appendix B.[20] In the empirical implementation, sales y_{ot} measures the sales of product categories by stores that belong to the same five-digit subsector for which we do not have product information in the local market m.[21] We include only local market variables in $\mathbf{x}_{jt}$ (e.g., population, population density, and income) and therefore use the notation $\mathbf{x}_{mt}$ instead of $\mathbf{x}_{jt}$ in what follows. The observed and unobserved product characteristics are aggregated at the store level using $\tilde{\alpha}_i$ as weights. The variable μ_{jt} is the weighed sum of all product demand shocks μ_{ijt} at the store level. Each store observes the demand shocks μ_{jt} when making input decisions, but their evolution is not under the store's control. Demand shocks related to product quality, location, checkout speed, the courteousness of store employees, parking, bagging services, cleanliness, etc. are part of μ_{jt}. In other words, demand shocks μ_{jt} include factors related to customer satisfaction and the quality of shopping in store j in period t.

The multiproduct sales-generating function (5) differs from a single product function by controlling for the impact of "competition" inside the store, which is represented by the effect of sales of other product categories on the sales of a product category in a store. By using the sales of different products in equation (5), we reduce the number of parameters to be estimated for multiproduct technology and obtain information on economies of scope. Therefore, we estimate only the coefficient of sales of products other than product i in store j (α_y) and not all coefficients α_i, $i = \{1, \cdots, np_j\}$. The coefficient α_y plays a key role in both the persistence in and level of productivity. The input coefficients in the multiproduct sales-generating function (5), i.e., β_l, β_k, β_a, β_q, are functions of the elasticity of substitution σ and are similar to the aggregate sales-generating function at the store (firm) level, which allows us to compare them with the estimates for a single-output technology.[22] In service industries, it is difficult to define

[20] The equation (5) is derived by rewriting the linear sum of product category sales $\sum_{i=1}^{np_{jt}} \left[\tilde{\alpha}_i y_{ijt} + \left(1 - \frac{1}{\sigma}\right) \tilde{\alpha}_y Y_{ijt}\right] \equiv \alpha_i y_{ijt} + \alpha_y y_{-ijt}$ and normalizing $\alpha_i = 1$.

[21] If the outside option is "do not buy," y_{ot} represents total sales in market m (aggregate sales). We show in Appendix B how to derive y_{ot} using the price equation and multiproduct technology.

[22] The coefficients of the multiproduct sales technology are functions of σ, i.e., $\beta_q = 1/\sigma$, $\beta_l = \tilde{\beta}_l(1-1/\sigma)$, $\beta_k = \tilde{\beta}_k(1-1/\sigma)$ and $\beta_a = \overline{\beta}_a(1-1/\sigma)$. Parameters σ_a and $\tilde{\beta}_a$ are included in $\overline{\beta}_a$, and they cannot be separately identified. Thus, we will not be able to identify separately the effect on inventory on supply and demand, that is, we identify the net effect through β_a (see the identification section and Appendix B).

a clean measure of technical productivity due to the complexity of measuring output and economies of scale and scope (Oi, 1992). Estimating only one coefficient for the other product categories (α_y) when controlling for prices has a cost – we cannot obtain a clean measure of technical productivity $\tilde{\omega}_{jt}$ because the coefficients of labor, capital and inventories include demand shocks even if we control for the elasticity of substitution. Therefore, the variable $\omega_{jt} \equiv (1 - 1/\sigma)\tilde{\omega}_{jt}$ measures revenue (sales) productivity. We simply refer to ω_{jt} as store productivity in what follows. The productivity measure ω_{jt} might include sales shocks due to approximations in (5), but all these sales shocks are different from demand shocks μ_{jt} that affect consumer preferences for product categories in a store. Nevertheless, productivity shocks ω_{jt} can be separated from the store's demand shocks μ_{jt}, which are part of the demand and affect store market share.

A few aspects about the multiproduct sales-generating function should be noted. First, store productivity and demand shocks affect sales, and they are not observed by the researcher but are observed by stores when decisions are made. Second, the multiproduct setting in Section 3.1 requires a positive α_y for static profit maximization to hold. This condition also holds in a dynamic setting because a policy function (input choice) should be optimal in each period.[23] Therefore, we now discuss the store's dynamic optimization problem and store's decisions that are used to recover ω_{jt} and μ_{jt}.

Stores' decisions. Stores compete in the product market and collect their payoffs. At the beginning of each time period, the incumbents decide whether to exit or continue to operate in the local market. Stores are assumed to know the scrap value they will receive upon exit δ prior to making exit and investment decisions.[24] If the store continues, it chooses the optimal levels of labor l (the number of employees), investment i, product variety np (the number of product categories), products bought from the wholesaler and inventory a. Store j maximizes the discounted expected value of future net cash flows using the Bellman equation:

$$\begin{aligned} V(\mathbf{s}_{jt}) = \quad & max\left\{\delta, \max_{np_{jt},a_{jt},l_{jt},i_{jt}} [\pi(\mathbf{s}_{jt}; np_{jt}, a_{jt}, l_{jt}, i_{jt}) - c_l(l_{jt}) \right. \\ & \left. -c_n(np_{jt}, a_{jt}, r_{mt}) - c_i(i_{jt}, k_{jt}) + \beta\mathbb{E}[V(\mathbf{s}_{jt+1})|\mathcal{F}_{jt}]]\right\}, \end{aligned} \tag{6}$$

where $\mathbf{s}_{jt} = (\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, np_{jt-1}, w_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt})$; r_{mt} measures the entry regulation in local market m in period t; w_{jt} is the logarithm of average wage at store j; $\pi(\mathbf{s}_{jt})$ is the profit function that is increasing in ω_{jt}, μ_{jt}, and k_{jt}; $c_l(l_{jt})$ is the labor cost; and $c_n(np_{jt}, a_{jt}, r_{mt})$ is the adjustment costs in product variety, which is increasing in inventory at the beginning of period n_{jt} and is affected by regulation r_{mt} (Joskow and

[23] Notably, the sign conditions on the first and the second derivatives that are used to prove Theorem 1 and Proposition 1 remain the same in a dynamic setting.

[24] The exit decision is included in the model to control for possible selection bias. However, we do not explicitly model the exit rule in the empirical application because we have fewer exits in our data.

Rose, 1989; Maican and Orth, 2018). For example, a more restrictive entry regulation increases stores' operating costs as a result of an increase in the fixed costs due to, for example, more expensive location or building costs, which affect stores' adjustment costs (see Section 3.3). Furthermore, $c_i(i_{jt}, k_{jt})$ is the investment cost of new capital (equipment), which is increasing in investment i_{jt} and decreasing in current capital stock k_{jt} for each fixed i_{jt} (Pakes, 1994);[25] β is a store's discount factor; and $\mathcal{F}_{jt}$ represents the information available at time t. Inventory holdings and investments in technology have dynamic implications due to adjustment costs, and both ω_{jt} and μ_{jt} are important for such adjustments.

The solution to a store's maximization problem (6) yields the optimal policy functions for the number of products $np_{jt} = \tilde{np}_t(\mathbf{s}_{jt})$, the sum of the store's inventories at the beginning of the period and the cost of products purchased $a_{jt} = \tilde{a}_t(\mathbf{s}_{jt})$, investments in technology $i_{jt} = \tilde{i}_t(\mathbf{s}_{jt})$, and exit $\chi_{jt+1} = \tilde{\chi}_t(\mathbf{s}_{jt})$.[26] We assume that labor $l_{jt} = \tilde{l}_{jt}(\mathbf{s}_{jt})$, which is part of profits $\pi(\cdot)$, is chosen to maximize short-run profits (Levinsohn and Petrin, 2003; Doraszelski and Jaumandreu, 2013; Maican and Orth, 2015; Maican and Orth, 2017).[27]

ASSUMPTION 3: *The store information set $\mathcal{F}_{jt}$ includes only current and past information on productivity, demand shocks, product variety in the previous period, input prices, and local market characteristics (not future values), for example, $\{\omega_{j\tau}, \mu_{j\tau}, np_{j\tau-1}, w_{j\tau}, k_{j\tau}, n_{j\tau}, y_{o\tau}, \mathbf{x}_{m\tau}, r_{m\tau}\}_{\tau=0}^{t}$. The remaining service output shocks u^p_{ijt} satisfy $E[u^p_{ijt}|\mathcal{F}_{jt}] = 0$.*

ASSUMPTION 4: *Store productivity and demand shocks follow two first-order Markov processes: (i) an endogenous process: $P_\omega(\omega_{jt}|\omega_{jt-1}, \mu_{jt-1}, r_{mt-1})$, where r_{mt-1} measures regulation in local market m in period $t-1$, (ii) an exogenous process: $P_\mu(\mu_{jt}|\mu_{jt-1})$, and (iii) the distributions $P_\omega(\cdot)$ and $P_\mu(\cdot)$ are stochastically increasing in ω and μ, and they are known to stores.*

Assumption 3 states that stores know their productivity ω_{jt}, demand shocks μ_{jt}, and local market conditions when they make decisions regarding their inputs, inventory, investments, and exit. Assumption 4 states that the demand shocks μ_{jt} are correlated over time according to a first-order Markov process

$$\mu_{jt} = h_t^\mu(\mu_{jt-1}; \gamma^\mu) + \eta_{jt}, \tag{7}$$

[25] In the empirical implementation, the main focus is on the adjustment cost in product variety. Therefore, to decrease computational complexity, we do not estimate adjustment costs in technology stock and labor.

[26] The exit rule χ_{jt} depends on the threshold productivity $\underline{\omega}_{mt}$, which is a function of all state variables except store productivity (Olley and Pakes, 1996). A store continues ($\chi_{jt} = 1$) if its productivity is larger than the local market threshold ($\omega_{jt} > \underline{\omega}_{mt}$).

[27] If labor has dynamic implications (e.g., in the case of labor adjustment costs), then labor in the previous period is part of the state space, and the optimal policy function for labor $l_{jt} = \tilde{l}_t(\mathbf{s}_{jt})$ is derived from solving the dynamic optimization problem (6).

where $h_t^{\mu}(\cdot)$ is an approximation of the conditional expectation and η_{jt} are shocks that are mean-independent of all information known at $t-1$.

Store productivity ω_{jt} follows an endogenous first-order Markov process where productivity, previous demand shocks, and entry regulation affect future productivity:

$$\omega_{jt} = h_t^{\omega}(\omega_{jt-1}, \mu_{jt-1}, r_{mt-1}; \gamma^{\omega}) + \xi_{jt}, \tag{8}$$

where $h_t^{\omega}(\cdot)$ is an approximation of the conditional expectation and ξ_{jt} are shocks to productivity that are mean-independent of all information known at $t-1$.[28] Stores can improve productivity after more intense competition from a less restrictive entry regulation and by using demand shocks. To survive fiercer competition after entry, incumbents improve productivity by learning practices from entrants (external learning). Stores can also use information about previous demand shocks, capturing why consumers choose a store, to improve productivity. For example, rearranging the products on the shelves such that consumers have faster access improves the store's efficiency in allocating resources.

ASSUMPTION 5: *Capital stock is a dynamic input that accumulates according to* $K_{jt+1} = (1-\delta_K)K_{jt} + I_{jt}$*, where* δ_K *is the depreciation rate. The investment level* I_{jt} *is chosen in period* t *and affects the firm in period* $t+1$*. The inventory level in period* $t+1$ *evolves according to* $N_{jt+1} = \tilde{N}_t(A_{jt}, Y_{jt})$*, where* A_{jt} *is the adjusted inventory, i.e., the inventories in the beginning of period* N_{jt} *adjusted by the products bought in period* t*. The function* $\tilde{N}_t(\cdot)$ *is increasing in* A_{jt} *and decreasing in* Y_{jt}*.*[29]

Inventory affects stores' service output because high inventory is costly to keep in stock and low inventory reduces consumers' choices. Products bought from wholesalers are an input that together with inventory at the beginning of period t (i.e., A_{jt}) lead to inventory levels in the beginning of period $t+1$ after realization of sales in period t (i.e., N_{jt+1}). Stores with high μ_{jt} increase their products bought from wholesalers. However, this also leads to a drop in inventories at the beginning of the next year because of the unexpected increase in sales. In other words, there is a distinction between how μ_{jt} affects current inventories and products bought from the wholesaler and the realization of inventories at the end of the year/start of next year.[30]

[28] It is straightforward to control for selection as in Olley and Pakes' (1996) framework by adding $\mathscr{P}_{jt}$ as an additional variable of $h_t^{\omega}(\cdot)$ function, where $\mathscr{P}_{jt}$ are predicted survival probabilities of being in the data in period t, conditional on the information in $t-1$, $\mathscr{P}_{jt} = Pr(\chi_{jt} = 1|\mathfrak{I}_{jt-1})$. The Markov process (8) implies that store productivity should shift, and stores that cannot improve productivity have to exit.

[29] If the variables are measured in physical units, inventory level in period $t+1$ evolves according to $N_{jt+1} = A_{jt} - Y_{jt}$.

[30] Cachon and Olivares (2010) argue that differences in store level inventory can arise because of differences in demand and competition. Lower margins decrease inventories, while a large choice set for consumers raises inventories. In addition, service production in the store can also drive differences in inventory across stores.

We now turn to the assumptions on the policy functions (input demand functions) that are required to recover productivity ω_{jt} and demand shocks μ_{jt}.

ASSUMPTION 6: *The labor demand function $l_{jt} = \tilde{l}_t(\mathbf{s}_{jt})$ is strictly increasing in ω_{jt}. The store's input product function $a_{jt} = \tilde{a}_t(\mathbf{s}_{jt})$ is strictly increasing in demand shocks μ_{jt}. The store productivity ω_{jt} and demand shocks μ_{jt} are part of the state space, i.e., $\omega_{jt}, \mu_{jt} \in \mathbf{s}_{jt}$, and the multivariate function $(\tilde{l}_{jt}, \tilde{a}_{jt})$ is a bijection onto (ω_{jt}, μ_{jt}).*

Assumption 6 is not restrictive and likely holds in many data sets. The most important assumption for a policy function to be consistent with the Bellman equation is to be strictly monotonic in the state variables. First, that productivity is increasing in labor can be shown when using Cobb-Douglas technology (Doraszelski and Jaumandreu, 2013; Maican and Orth, 2015; Maican and Orth, 2017).[31] This characteristic implies that more productive stores do not have disproportionately higher markups than less productive stores. In addition, the fact that the inventory demand function is increasing in demand shocks received by stores is valid in retail markets. Maican and Orth (2017) show that an input demand function is strictly increasing in productivity under imperfect competition when the marginal product of the input is increasing in productivity, which is fully consistent with store profit maximization behavior. Second, in our case with two unobservables, the invertibility implies solving systems of nonlinear equations. A key condition for invertibility is that the determinant of the Jacobian is not zero. This condition is satisfied when productivity and demand shocks have different impacts on labor and inventory and the relative impact is not the same $(\partial\tilde{l}/\partial\omega)/(\partial\tilde{l}/\partial\mu) \neq (\partial\tilde{a}/\partial\omega)/(\partial\tilde{a}/\partial\mu)$. This requirement is not restrictive and can be empirically tested using the estimated policy functions (see Section 4). Appendix C discusses in greater detail the invertibility of the system of equations in our model.

Market share index function. Following the recent developments from the production function literature to control for unobservables, we use an output index function and an input process to recover the demand shocks μ_{jt} (Ackerberg et al., 2007). Store demand shocks μ_{jt} are defined as a weighted sum of product category-specific demand shocks of store j from the demand system (4) and include information that affects consumers' store choice and the store's market share. Most importantly, the aggregation weights in μ_{jt} arise from the multiproduct service technology (1). Thus, the store's market share is an informative output for the index function, which is computed using product-category sales that are affected by demand shocks. We use inventory before sales, as it contains information about μ_{jt}, as input demand. A complication of using a store-level aggregate demand system, where consumers obtain utility from choosing a store, is the need for price data and defining a basket of products to calculate a price in-

[31] This assumption holds in our case because the transcendental technology is a generalization of Cobb-Douglas, that is, it has Cobb-Douglas technology in inputs but not in outputs.

dex consistent with the multiproduct service technology.[32] Indeed, we rely on all stores in five-digit service industries for which price data are scant. Even though one would access price data, it is difficult to define an annual price index, given that labor and capital are observed yearly.

To be consistent with multiproduct sales, the index function needs to satisfy the following properties. First, it aggregates stores' category sales from the multiproduct sales function in the output index τ_{jt} and is informative about store demand and is consistent with aggregate demand in a local market (i.e., it includes μ_{jt}). Second, to help identification the index function allows μ_{jt} to appear additively. The main aim of the index function is to identify μ_{jt} separately from ω_{jt} and not to infer, e.g., changes in price elasticities due to repositioning in product categories. Third, the index function together with multiproduct sales enables us to compute sales in the outside option and therefore total sales in a local market after changes in the local environment. We consider the output of an index function with store and market characteristics $\boldsymbol{\delta}_{jt}$ (that can include $\mathbf{x}_{mt}$) and μ_{jt} as arguments

$$\tau_{jt} = \tau_t(\boldsymbol{\delta}_{jt}; \boldsymbol{\rho}) + \mu_{jt} + \nu_{jt}, \tag{9}$$

where the output index $\tau_{jt} = ln(ms_{jt}) - ln(ms_{0t})$ is the ratio of the store market share and the market share of the outside option, ms_{jt} is the market share of store j in local market m in period t computed at the five-digit industry sector level using sales, ms_{0t} is the outside option, i.e., the market share of other stores in market m computed at the five-digit industry sector level (we have the same outside option as in equation (5), but here we use a share-based measure), and ν_{jt} is an error term that is mean-independent of all controls. In the empirical implementation, we choose a simple linear specification for $\tau_t(\cdot)$, i.e., $\tau_t(\boldsymbol{\delta}_{jt}; \boldsymbol{\rho}) = \rho_{np} np_{jt} + \rho_{inc,1} inc_{mt} + \rho_{inc,2} inc_{mt}^2$, where inc_{mt} is the logarithm of the average income in the local market.

We now explain the importance of the market share index function and its link to multiproduct sales technology. First, services frequently rely on sales that depend on both demand and supply to measure output. In our model, sales depend on both the store's demand shocks μ_{jt} and productivity ω_{jt}, whereas a store's market share index function depends only on μ_{jt}. To guarantee consistency and identification of our model, the demand shocks μ_{jt} connect the market share index function (9) and the sales-generating function (5). Because the sales-generating function (5) controls for capital stock k_{jt} and inventory a_{jt}, they are not part of μ_{jt}, and we do not need to control

[32] As in a nested-logit model, we can use the demand system and derive the probability of choosing store j as a function of p_{ijt} and μ_{ijt} using the conditional choice probability. However, this is not helpful in the identification because p_{ijt} and μ_{ijt} are not observed.

for them in the market share index function.[33] The number of product categories np_{jt} affects a_{jt}, which includes additional information such as the volume of each product, and products are aggregated based on monetary value.

Second, supply-side weights included in μ_{jt} and remaining shocks ν_{jt} restrict us from relying on nonparametric inversion from the discrete choice literature to recover μ_{jt}. Although the market share index function (9) is not a logit demand specification, being a function of np_{jt} and ν_{jt} but not the price, it is useful for understanding store local market demand. The market share index function uses the same output as a logit demand consistent with CES assumptions. The reason is that market share captures information about local market demand and enables a simple expression from the logarithm of store sales and the outside option.[34]

Third, we obtain a joint system of equations from the multiproduct sales equation and the market share index function allowing solution using the nested fixed-point algorithm. We have two systems of equations: sales per product category at the store level (equation (5)) and the store local market share (equation (9)). Using recovered demand shocks, we solve the joint systems of equations to obtain sales per product category and the outside option local market sales (total sales) following policy interventions that affect stores' primitives.[35]

Numerical implementation. We describe how the estimated model can be used to compute changes in sales per product category and sales of stores in the outside option (y_{ot}) after policy changes. A numerical implementation of the model also helps improve the understanding of the integration of different parts of the model. For simplicity of exposition, we assume to have only one store ($j = 1$) for which we observe the number of product categories and have recovered productivity and demand shocks by estimating the model. The multiproduct sales equation can be written as $y_{i1t} = -\alpha_y y_{-i1t} + (1/\sigma)y_{ot} + T_{1t} + \mu_{1t}$, where term T_{1t} groups all store characteristics (labor, capital, inventory, productivity, and market characteristics) that are in equation (5), and $i = \{1, \cdots, np_1\}$ indexes the product categories of the store. The market share index equation can be written as $ln(\sum_{i=1}^{np_1} exp(y_{i1t})) - ln(y_{ot}) = \boldsymbol{\delta}_{1t}\boldsymbol{\rho} + \mu_{1t}$. We start with an initial value for y_{ot} denoted by $y_{ot}^{(0)}$. Then, we use the multiproduct equa-

[33] Even if we control for capital stock k_{jt} and inventory a_{jt} in the market share index equation, we cannot separately identify their effects on demand and supply; i.e., we identify the net effect. Appendix B presents a short discussion of identification of β_a.

[34] The ratio of market shares of two stores depends on the number of product categories they offer and demand shocks. Because store-specific demand shocks depend on the product-category mix, the market share ratio changes if one of the stores alters its product-category mix without changing the number of product categories. Nevertheless, one way to avoid the IIA problem specific to logit models in equation (9) is to group product categories by a store characteristic and rewrite equations (4) and (9) in a nested-logit form. However, this is beyond the aim of this paper.

[35] The market share index function is not useful in counterfactuals if we assume that the outside option sales are unaffected by changes in the local environment. Therefore, the index function is used only in identification to recover demand shocks μ_{jt}.

tion to compute sales per product category $y_{i1t}^{(0)}$ using the fixed-point algorithm to solve the multiproduct sales system of equations (the number of equations is given by the number of categories). The computed sales per product category $y_{i1t}^{(0)}$ are used to obtain the next sales of the outside option $y_{ot}^{(1)}$, which are used to compute next period's sales per product category $y_{i1t}^{(1)}$. We repeat this process until $\|y_{i1t}^{(n+1)} - y_{i1t}^{(n)}\| < tol$ and $\|y_{ot}^{(n+1)} - y_{ot}^{(n)}\| < tol$, where tol is a numerical tolerance level, and n is the number of iterations. The same algorithm is applied if there are many stores in a market for which we observe their product categories.[36]

Equilibrium. Local and sectorial policies affect stores' costs, inventory, investment in technology, labor and exit. We assume that these policies are unexpected and permanent once they are implemented.

ASSUMPTION 7: *The equilibrium in the industry is stationary and is given by the Markov Perfect Equilibrium [MPE], which includes the policies* $\tilde{np}_t(\mathbf{s}_{jt})$, $\tilde{a}_t(\mathbf{s}_{jt})$, $\tilde{l}_t(\mathbf{s}_{jt})$, $\tilde{i}_t(\mathbf{s}_{jt})$, $\tilde{\chi}_t(\mathbf{s}_{jt})$ *and the value function* $V(\mathbf{s}_{jt})$ *that are consistent with stores' optimization problem (6).*

Thus, to form expectations, stores use the optimal policies. The MPE equilibrium implies that the state variables satisfy the Markov property before and after a change in regulation or another policy. Conditional on the state variables, the stationarity of the equilibrium implies that the value functions are not indexed by time.

3.2.1 Identification and estimation

The identification and estimation of the sales-generating function, including the Markov processes for ω_{jt} and μ_{jt}, are based on the well-established two-step methods in the production function literature. Identification comes from a system of equations (multiproduct sales and market share) and two unobservables (productivity and demand shocks), where one of the unobservables is part of only one equation. Two control functions based on the store's optimal policy functions are used to proxy for ω_{jt} and μ_{jt}.[37]

We estimate β_l, β_k, β_a, α_y, σ, ρ_{np}, $\rho_{inc,1}$, $\rho_{inc,2}$, γ^{ω}, and γ^{μ} together using a modified Olley and Pakes (1996) (OP) two-step estimator that includes product information (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg et al., 2015; Gandhi et al., 2020). Compared to OP, we have two unobservables to recover, and we show how the market share index function helps to recover demand shocks μ_{jt} separate from productivity ω_{jt} and ensures the identification of the model.[38] In our retail setting, the

[36]The simulations demonstrate a fast convergence of the algorithm. The authors provide results of Monte Carlo simulations in Julia upon request for a large number of products and stores.

[37]Ackerberg et al. (2007) (Section 2.4) and Matzkin (2008) discuss the core of identification of such system of equations.

[38]Maican et al. (2020) estimate impact of R&D investments on domestic and foreign sales in Sweden. Using a single product function setting and a system of two equations (domestic and export sales), they recover domestic and foreign revenue productivities from investment demand and the number of export

dynamics of store productivity are more complex, since productivity is affected by both demand shocks and local entry regulations.

Recovering productivity and demand shocks. The general labor demand and inventory functions that arise from the stores' optimization problem (6) are

$$\begin{aligned} l_{jt} &= \tilde{l}_t(\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt}) \\ a_{jt} &= \tilde{a}_t(\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt}) \end{aligned}$$

To back out ω_{jt} and μ_{jt}, assumption 6 must hold; i.e., the functions $\tilde{l}_t(\cdot)$ and $\tilde{a}_t(\cdot)$ must be strictly monotonic in ω_{jt} and μ_{jt}, which holds under mild regularity conditions on the dynamic programming problem (6).[39] Stores react to high demand shocks μ_{jt} by increasing inventories without changing product categories (i.e., higher love-for-variety), which implies more inventory. Technological advances in the store can benefit the existing number of product categories through faster product lines and a higher frequency of turnover (Holmes, 2001). Higher productivity also creates incentives for stores to increase their product variety and store size. By inverting these policy functions to solve for ω_{jt} and μ_{jt}, we obtain

$$\begin{aligned} \omega_{jt} &= f_t^1(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt}) \\ \mu_{jt} &= f_t^2(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt}), \end{aligned} \qquad (10)$$

which yields that the productivity and exogenous shocks are nonparametric functions of the observed variables in the state space and the controls.

The estimation of the sales-generating function (5) and the market share index equation (9) is done together in two steps. In the first step, we construct measures of productivity ω_{jt} and demand shocks μ_{jt} as functions of the structural parameters that do not include the remaining shocks u_{ijt}^p and ν_{jt}. To do this, we use equations (5) and (9) and the solution of the system of nonparametric policy functions given by (10).

By substituting the nonparametric inversion $f_t^2(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt})$ for μ_{jt} in (9) and considering that the number of product categories np_{jt} is also a function of the store state variables (a policy function of the store optimization problem), the market share equation can be written as $ln(ms_{jt}) - ln(ms_{0t}) = b_t(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt}) + \nu_{jt}$, which can be estimated using ordinary least squares (OLS) and a polynomial expansion of order two in l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, $\mathbf{x}_{mt}$, r_{mt} to approximate function $b_t(\cdot)$.[40] Therefore, we obtain an estimate of $b_t(\cdot)$, denoted $\hat{b}_t$, which is the predicted $ln(ms_{jt}) - ln(ms_{0t})$. This allows us to write demand shocks μ_{jt} as a parametric func-

destinations functions.

[39] See Appendix C, Pakes (1994).

[40] A polynomial expansion of order three shows no improvement in the estimation of the first stage. Other approximations can be used, such as b-splines, for example.

tion: $\mu_{jt} = \hat{b}_{jt} - \rho_{np} np_{jt} - \rho_{inc,1} inc_{mt} - \rho_{inc,2} inc_{mt}^2$, which will be treated as an input in the multioutput sales-generating function (5).

In the second step, by substituting μ_{jt} (predicted) and ω_{jt} into (5), the sales-generating function becomes

$$y_{ijt} = -\alpha_y y_{-ijt} + \phi_t(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt}) + u_{ijt}^p, \tag{11}$$

where $\phi_t(\cdot) = \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \beta_q y_{ot} + \mathbf{x}_{mt}' \boldsymbol{\beta}_x + \omega_{jt} + \mu_{jt}$. The function $\phi_t(\cdot)$ can be approximated using a polynomial expansion of order two in its arguments. The estimation of (11) yields an estimate of service output without service output shocks u_{ijt}^p, which gives us $\hat{\phi}_t$, which is used to obtain store productivity ω_{jt} as a function of the parameters, $\omega_{jt} = \hat{\phi}_{jt} - \beta_l l_{jt} - \beta_k k_{jt} - \beta_a a_{jt} - \beta_q y_{ot} - \mathbf{x}_{mt}' \boldsymbol{\beta}_x - \hat{b}_{jt} + \rho_{np} np_{jt} + \rho_{inc,1} inc_{mt} + \rho_{inc,2} inc_{mt}^2$.

Then, we rewrite the sales and market share equations using parametric forms of productivity ω_{jt} and demand shocks μ_{jt} and Markov processes

$$\begin{aligned} y_{ijt} = \;& -\alpha_y y_{-ijt} + \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \beta_q y_{ot} + \mathbf{x}_{mt}' \boldsymbol{\beta}_x + \hat{b}_{jt} - \rho_{np} np_{jt} \\ & -\rho_{inc,1} inc_{mt} - \rho_{inc,2} inc_{mt}^2 + h^{\omega}(\hat{\phi}_{jt-1} - \beta_l l_{jt-1} - \beta_k k_{jt-1} - \beta_a a_{jt-1} \\ & -\beta_q y_{ot-1} - \mathbf{x}_{mt-1}' \boldsymbol{\beta}_x - \hat{b}_{jt-1} + \rho_{np} np_{jt-1} + \rho_{inc,1} inc_{mt-1} \\ & +\rho_{inc,2} inc_{mt-1}^2, \hat{b}_{jt-1} - \rho_{np} np_{jt-1} - \rho_{inc,1} inc_{mt-1} - \rho_{inc,2} inc_{mt-1}^2, \\ & r_{mt-1}) + \xi_{jt} + u_{ijt}^p \end{aligned} \tag{12}$$

$$\begin{aligned} ln(ms_{jt}) - ln(ms_{0t}) = \;& \rho_{np} np_{jt} + \rho_{inc,1} inc_{mt} + \rho_{inc,2} inc_{mt}^2 + h^{\mu}(\hat{b}_{jt-1} \\ & -\rho_{np} np_{jt-1} - \rho_{inc,1} inc_{mt-1} - \rho_{inc,2} inc_{mt-1}^2) \\ & +\eta_{jt} + \nu_{jt}. \end{aligned} \tag{13}$$

The parameters of the multiproduct sales function (12) and market share equation (13) are identified using moment conditions on the remaining shocks in these equations, $\xi_{jt} + u_{ijt}^p$ and $\eta_{jt} + \nu_{jt}$.

Estimation. In the empirical implementation, we approximate the functions $h^{\omega}(\cdot)$ and $h^{\mu}(\cdot)$ in the Markov processes of ω_{jt} and μ_{jt} by polynomials. The estimated Markov processes are:

$$\begin{aligned} \omega_{jt} = \;& \gamma_0^{\omega} + \gamma_1^{\omega} \omega_{jt-1} + \gamma_2^{\omega} (\omega_{jt-1})^2 + \gamma_3^{\omega} (\omega_{jt-1})^3 + \gamma_4^{\omega} \mu_{jt-1} + \gamma_5^{\omega} r_{mt-1} \\ & +\gamma_6^{\omega} \omega_{jt-1} \times \mu_{jt-1} + \gamma_7^{\omega} r_{mt-1} \times \omega_{jt-1} + \gamma_8^{\omega} r_{mt-1} \times \mu_{jt-1} + \xi_{jt} \end{aligned} \tag{14}$$

$$\mu_{jt} = \gamma_0^{\mu} + \gamma_1^{\mu} \mu_{jt-1} + \gamma_2^{\mu} (\mu_{jt-1})^2 + \gamma_3^{\mu} (\mu_{jt-1})^3 + \eta_{jt} \tag{15}$$

We denote by $\boldsymbol{\theta}$ the vector of parameters to be estimated, $\boldsymbol{\theta} = (\beta_l,\ \beta_k,\ \beta_a,\ \alpha_y,\ \sigma,\ \boldsymbol{\beta}_x,\ \rho_{np},\ \rho_{inc,1},\ \rho_{inc,2},\ \boldsymbol{\gamma}^{\omega},\ \boldsymbol{\gamma}^{\mu})$. Productivity ω_{jt} and μ_{jt} are functions of $\boldsymbol{\theta}$. We can identify $\boldsymbol{\theta}$ coefficients using moment conditions based on $(\xi_{jt} + u^p_{ijt})$ and $(\eta_{jt} + \nu_{jt})$ and the generalized method of moments (GMM) estimator. The identification uses that the current shocks are conditionally independent from information in $t-1$, $\mathcal{F}_{jt-1}$.[41] Thus, to identify $\boldsymbol{\theta}$ we use the moment conditions $E[\xi_{jt} + u^p_{ijt}|\ y_{-ijt-1},\ l_{jt-1},\ k_{jt-1},\ a_{jt-1},\ \mathbf{x}_{mt-1}] = 0$ and $E[\eta_{jt} + \nu_{jt}|np_{jt-1}, inc_{mt-1}, inc^2_{jt-1}] = 0$. The parameters β_l, β_k, and β_a are identified using l_{jt-1}, k_{jt-1}, and a_{jt-1} as instruments. Thus, we use that the current remaining productivity and sales shocks are not correlated with previous inputs to form moment conditions.

To identify the economies of scope parameter α_y, we use y_{-ijt-1} as an instrument, which requires that the previous output is not correlated with current remaining sales and productivity shocks. Monte Carlo experiments discussed below show the robustness of the identification of the scope parameter α_y using previous output.[42] That previous local market characteristics $\mathbf{x}_{mt-1}$ are not correlated with current sales and productivity shocks allows us to identify $\boldsymbol{\beta}_x$.[43] To identify the coefficients of the market share equation, we use that $(\eta_{jt} + \nu_{jt})$ are not correlated with the previous number of product categories and income. The Markov process parameters $\boldsymbol{\gamma}^\omega$ and $\boldsymbol{\gamma}^\mu$ are identified using the corresponding polynomial terms in equations (14) and (15) as instruments.

The parameters $\boldsymbol{\theta}$ are estimated by minimizing the GMM objective function

$$\min_{\boldsymbol{\beta}} Q_N = \left[\frac{1}{N} W' v(\boldsymbol{\theta})\right]' A \left[\frac{1}{N} W' v(\boldsymbol{\theta})\right], \tag{16}$$

where $v_{jt} = (u_{ijt} + \xi_{jt}, \nu_{jt} + \eta_{jt})'$, W is the matrix of instruments, and A is the weighting matrix defined as $A = \left[\frac{1}{N} W' v(\boldsymbol{\beta}) v'(\boldsymbol{\beta}) W\right]^{-1}$. Standard errors are computed according to Ackerberg et al. (2012).[44]

Monte Carlo simulations. We provide Monte Carlo simulations to show the identification of multioutput technology with two inputs labor and capital using the control function approach.[45] We also discuss the bias of labor and capital coefficients of a single-output technology when the true data generating process (DGP) of total output is a multiproduct technology.

The multiproduct technology is estimated at the product level assuming the same

[41] Ackerberg et al. (2007) and Wooldridge (2009) discuss the use of previous variables as instruments in a two-step control function approach when estimating production technologies. As Ackerberg et al. (2015) discuss in Section IV(i), there are many ways to estimate an Olley and Pakes' framework based on second step moments. Most important, stronger assumptions can lead to more precise estimates. Our empirical results remain robust using moment conditions based on ξ_{jt} and η_{jt} to identify parameters β_l, β_k, β_a, $\boldsymbol{\beta}_x$, β_q, ρ_{np}, $\rho_{inc,1}$, and $\rho_{inc,2}$ in the empirical application.

[42] In the next subsection, we also discuss an alternative estimator that is computationally more demanding.

[43] In general, $\mathbf{x}_{mt}$ can also be used as instruments because market characteristics are exogenous.

[44] Bootstrapping might not be the best choice when the underlying model is more complicated.

[45] We present a short summary based on the results in Maican and Orth (2019).

production technology across products.[46] We focus on a simple specification and assume perfect competition; therefore, we choose $y_{ijt} = \alpha_y y_{-ijt} + \beta_l l_{jt} + \beta_k k_{jt} + \omega_{jt} + u_{jt}$. We consider 1,000 stores and set $\beta_l = 0.6$, $\beta_k = 0.4$, and $\alpha_y = -0.85$. Most of our simulation settings are similar to those used by the previous literature on production functions (e.g. Van Biesebroeck, 2007; Ackerberg et al., 2015). Productivity follows an $AR(1)$ process ($\omega_{jt} = \rho\omega_{jt-1} + \xi_{jt}$) with persistence $\rho = 0.7$. Productivity is simulated to have constant variance over time (standard deviation 0.3). Wages w_{jt} follow an $AR(1)$ process with persistence $\rho^w = 0.3$ and are simulated to have constant variance over time (standard deviation 0.3). Labor is simulated using the first-order condition of static profit maximization. Capital stock is constructed using the perpetual inventory method $K_{jt} = (1 - 0.2)K_{jt-1} + I_{jt-1}$.[47] The number of years (periods) is 10, and all variables are used in the steady state.[48]

To estimate α_y, β_l and β_k, we use a two-step estimator with labor demand as a proxy for store productivity (Olley and Pakes, 1996; Doraszelski and Jaumandreu, 2013; Ackerberg et al., 2015). The identification of $(\alpha_y, \beta_l, \beta_k)$ is based on the moment conditions $\mathbb{E}[\xi_{jt}|y_{-ijt-1}, l_{jt-1}, k_{jt}] = 0$ and GMM estimator. Table B.1 in Appendix B shows the estimates of the single- and multioutput technology based on 1,000 Monte Carlo simulations. For the multiproduct technology, each store has three products, and their outputs are obtained by solving the nonlinear system of equations for each store in each period. The findings in Table B.1 show that we identify the parameters without bias when the DGPs are the true ones (single- and multiproduct DGPs) even if the estimation uses nonparametric labor demand and the data are generated using parametric labor demand.

Table B.2 in Appendix B shows the bias in the labor and capital coefficients of a single-output technology when the true DGP is a multi-output technology with three products. The results show a downward biased labor coefficient (decrease from 0.6 to 0.49) and an upward biased capital coefficient (an increase from 0.4 to 0.542). These biases that translate into a large productivity bias are generated by the omission of the tradeoff between producing one product or different products with the same resources that affect the aggregate output. In a multiproduct setting, the productivity difference between two stores using the same inputs is generated by the choice of product mix.

[46] Again, this can be relaxed in an empirical setting.

[47] Investment is simulated based on a policy function that is increasing the in store's state variables, i.e., $i_{jt} = 0.2 + 0.3\omega_{jt} + 0.1k_{jt}$. For robustness, we have used a nonlinear specification $i_{jt} = 0.2 + 0.3\omega_{jt} + 0.1k_{jt} + 0.01\omega_{jt}^2 + 0.01k_{jt}^2 - 0.004\omega_{jt}^3 - 0.006k_{jt}^3$. However, because there are no substantial changes in the main findings, we show the results with the linear specification.

[48] We consider 100 warm-up simulations before simulating the data sets.

3.2.2 Discussion and extensions

Alternative identification for economies of scope parameter. There is an alternative identification strategy for the scope parameter α_y that fully endogenizes product-category sales in the estimation. That is, we can solve the system of output equations for each store instead of using the previous output of other product categories as an instrument. This is similar to the counterfactual experiments where we solve the system of output equations for each store. However, this estimator is computationally demanding because it requires to solve the system of equations for each store-year observation and a new set of model parameters using fixed-point iteration. Monte Carlo experiments show no main advantages of this alternative estimator over the above IV identification strategy when stores use the same sales technology for their product categories.

Endogeneity of regulation. Because stores cannot influence or form expectations about the future stringency of regulation, we follow a two-step estimation procedure to alleviate endogeneity concerns regarding regulation. Our estimation takes into account possible endogeneity of the regulation measure. We model the structure of supply and demand shocks and use many exogenous local market characteristics as controls in the first stage. Entry regulation is exogenous in the productivity process such that individual stores do not affect the outcome of regulation or form expectations about the stringency of future regulation. The nature of the semiparametric model helps address the possible endogeneity of regulation on productivity. Removing the effect of local market characteristics from the sum of demand and production shocks in the first step reduces endogeneity concerns when estimating the productivity process. If productivity shocks ξ_{jt} are correlated with the previous stringency of regulation, we can identify the coefficient of r_{mt-1} by using an instrument. Our instrument needs to be correlated with regulatory stringency but be unrelated to shocks in productivity ξ_{jt}. In the data section, we discuss the endogeneity of entry regulation using the instrumental variable (IV) approach and three instruments (Table 3).

Alternative demand specification. Our main empirical results are not driven by the demand assumption (the general form of sales-generating function remains the same when allowing for nests) and are supported by various simple descriptives and reduced-form specifications (see Section 2).

The simple demand approach in Section 3.2 has a key benefit: CES preferences generate the same demands as would be obtained from aggregating many consumers who make discrete choices over in what store to shop. On the other hand, CES preferences impose a specific structure on demand, which is restrictive. Nevertheless, our model is rich on the supply side and the form of our multiproduct sales-generating function (5) is also consistent with a demand specification that allows for rich substitution patterns, e.g., a constant expenditure specification in an aggregate nested logit model where price enters in log form. This is because in a constant expenditure specification, we use the volume of sales for each product category, which allows us to aggregate products when using the multiproduct function (3).[49] In a nested demand model, consumers choose stores and then products within a store. In this case, the output and input parameters depend on the nest(s) parameter(s) and the scope parameter α_y includes information about product correlation in the nests at the store level. We use the simple CES specification in the estimation because we do not focus on a specific product category in the empirical application (e.g., yogurt) and we have high heterogeneity on supply side in the data.

The relationship with other multiproduct estimators. Our model uses product output shares and store inputs from the data. There are also alternative estimators that estimate and use input shares to study multiproduct. In contrast to many alternative estimators of multiproduct, we explicitly model economies of scope into the technology and endogenize the number of products. Our model is closely related to De Loecker et al. (2016) even if their method estimates input shares to construct single product technology. As in De Loecker et al. (2016), we have separability in inputs and outputs in the production technology and model firm/store productivity and not product-firm productivity.[50] In the retail context, it is difficult to define a meaningful measure of product-store productivity. Using the aggregation over inputs and outputs, Maican and Orth (2019) show that there is a direct relationship between the input shares from a Cobb-Douglas technology at the product level and output shares of transcendental technology. The relationship exists because both technologies use firm/store productivity, and there is no need to aggregate product productivity to the firm level.

Separating input allocations per product can be difficult in service industries. For example, different machinery and equipment are used to carry or to store different product categories in the same time to increase efficiency. In the multiproduct case, a service technology function consistent with profit maximization implies aggregation over physical products, and this is restrictive for many data sets due to large heterogeneity (especially in retailing). The service sector is characterized only by multiproduct and, in many cases, it is also difficult to measure physical product. Splitting all inputs is not entirely consistent with economies of scale and scope in retail. Since our focus is on entry regulations and economies of scope and not recovering product markups, transcendental technology that uses observed output shares is preferable; it does not require additional assumptions to recover input shares (not observed in the data).

[49] All technical derivations are available from authors upon request. A constant expenditure specification allows consumers to buy more than one product (Verboven, 1996).

[50] See also Valmari (2016), Dhyne et al. (2017), and Orr (2018).

3.3 Estimation of the dynamic model

This subsection discusses the estimation of the dynamic model that is used to compute the optimal number of product categories and long-run profits after changes in entry regulations or other changes in the local business environment.

Regulation and adjustment costs of product variety. Entry regulations affect stores' operating costs. A more restrictive regulation can increase stores' operating costs through higher fixed costs (e.g., expensive location or building costs) (Joskow and Rose, 1989; Maican and Orth, 2018). In markets with fewer stores, the cost of logistics can increase and product differentiation decreases. Consumers in these markets need to travel longer distances and can compensate for the longer traveling time by spending less time in a store. Therefore, entry regulations impact the adjustment costs in product variety (and inventory) through both demand and supply channels.

We assume that stores have quadratic adjustment costs in product categories (millions SEK):

$$\begin{aligned} c_n(np_{jt}, a_{jt}, r_{mt}; \boldsymbol{\varphi}) = \;& \varphi_1 np_{jt} + \varphi_2 np_{jt}^2 + \varphi_3 exp(a_{jt})^2 + \varphi_4 exp(a_{jt}) np_{jt} \\ & + \varphi_5 np_{jt} r_{mt} + \varphi_6 exp(a_{jt}) r_{mt} \end{aligned} \tag{17}$$

The marginal effect of more liberal entry regulation (i.e., an increase in r_{mt}) on adjustment costs in variety depends on the store's number of product categories and size of inventory demand a_{jt}. A change in entry regulations affects the store's cost and its productivity and, therefore, the number of product categories and sales.

To reduce the computational complexity, we do not model the adjustment costs of labor and investment in the empirical application. Thus, the store's value function is given by the following Bellman equation

$$\begin{aligned} V(\mathbf{s}_{jt}) = \;& \max_{np_{jt}, a_{jt}} \{\pi_{jt}(\mathbf{s}_{jt}; np_{jt}, a_{jt}, l_{jt}) - c_n(np_{jt}, a_{jt}, r_{mt}; \boldsymbol{\varphi}) \\ & + \beta \mathbb{E}[V(\mathbf{s}_{jt+1}) | \mathcal{F}_{jt}]\}, \end{aligned} \tag{18}$$

where $\pi_{jt}(\mathbf{s}_{jt})$ measures the variables profits and $\boldsymbol{\varphi} = (\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6)$, are the parameters to be estimated in the dynamic stage using value function approximation and simulation (Ryan, 2012; Sweeting, 2013; Barwick et al., 2018; Maican, 2019).

We approximate the value function V using radial basis function networks (RBFN), i.e., $V(\mathbf{s}_{jt}) = \mathbf{bs}(\mathbf{s}_{jt})\boldsymbol{\kappa}$, where $\mathbf{bs}(\cdot)$ are the basis functions (Mai-Duy and Tran-Cong, 2003; Sutton and Barto, 2018). This allows us to rewrite the Bellman equation as

$$\begin{aligned} V(\mathbf{s}_{jt}; \boldsymbol{\kappa}) = \;& \max_{np_{jt}, a_{jt}} \{\pi_{jt}(\mathbf{s}_{jt}; np_{jt}, a_{jt}, l_{jt}) - c_n(np_{jt}, a_{jt}, r_{mt}; \boldsymbol{\varphi}) \\ & + \beta \mathbb{E}[V(\mathbf{s}_{jt+1}; \boldsymbol{\kappa}) | \mathcal{F}_{jt}]\}. \end{aligned} \tag{19}$$

For each set of cost parameters $\boldsymbol{\varphi}$, we use the linearity property of the value function approximation to find the approximation parameters $\boldsymbol{\kappa}$ such that the Bellman equation holds. We use the RBFN approximation to find the optimal policies using the state variables in t and $t+1$. The estimation of multiproduct technology gives the transitions for productivity and demand shocks. The inventory at the end of the period (i.e., beginning of next period) is estimated using a b-splines approximation in a_{jt} and y_{jt}.[51]

To compute total store-level sales using our model, we need to solve a system of nonlinear equations (for each store), which is given by the multiproduct technology. This system of equations has a unique solution and is solved using fixed-point iteration (Maican and Orth, 2019). The store's total sales are a function of the number of product categories. Net profits $\pi(\cdot)$ are computed as sales minus labor cost.

Estimation. Given an initial estimate of $\boldsymbol{\varphi}$ and approximation parameters $\boldsymbol{\kappa}$, we solve the first-order condition in the Bellman equation (19) to find the optimal number of product categories np_{jt} and inventory a_{jt} at each state. Then, new value function approximation parameters $\boldsymbol{\kappa}$ are found by solving the Bellman equation. The cost parameters are estimated using the method of indirect inference (Gourieroux and Monfort, 1996; Li, 2010). The estimator matches the percentiles of the observed number of product categories (np_{jt}) and inventory (a_{jt}) distributions $\mathcal{P}_x$ ($x = [.05, .10, .15, \cdots, .95]$) with percentiles generated by the policy functions from the model (i.e., solving the system of first-order conditions). We denote the vector moments generated by the model as $\tilde{\boldsymbol{\mathcal{P}}}(\boldsymbol{\varphi})$, which depend on the structural parameters, and $\boldsymbol{\mathcal{P}}$ as the corresponding vector of data moments. The criterion function minimizes the distance between the moments $\tilde{\boldsymbol{\mathcal{P}}}(\boldsymbol{\varphi})$ and $\boldsymbol{\mathcal{P}}$

$$J(\boldsymbol{\varphi}) = [\boldsymbol{\mathcal{P}} - \tilde{\boldsymbol{\mathcal{P}}}(\boldsymbol{\varphi})]' \boldsymbol{W} [\boldsymbol{\mathcal{P}} - \tilde{\boldsymbol{\mathcal{P}}}(\boldsymbol{\varphi})], \tag{20}$$

where $\boldsymbol{W}$ is a weighting matrix.[52] The cost function coefficients are identified by matching the observed and predicted percentiles of the distribution of np_{jt} and a_{jt}.[53] The standard errors are computed using subsampling.

[51]We assume that changes in the stringency of regulation do not affect the structural form of this relationship. However, regulation affects the variables of this function, a_{jt} and y_{jt}.

[52]The identity matrix is used in the empirical setting.

[53]The cost parameters can also be estimated using the inequality estimator that uses alternative policy $a'(\mathbf{s}_{jt}) = \hat{a}(\mathbf{s}_{jt}) + \psi^a$, $np'(\mathbf{s}_{jt}) = \hat{np}(\mathbf{s}_{jt}) + \psi^{np}$, where $\psi^a \sim N(0,1)$ and $\psi^{np} \sim \{-1, 1\}$ (see Bajari et al., 2007). Let c be any combination of $(\mathbf{s}_{jt}, a'_{jt}, np'_{jt})$, and define $m(c; \boldsymbol{\kappa}, \boldsymbol{\varphi}) = V(\mathbf{s}_{jt}; \boldsymbol{\kappa}, \boldsymbol{\varphi}, a_{jt}, np_{jt}) - V(\mathbf{s}_{jt}; \boldsymbol{\kappa}, \boldsymbol{\varphi}, a'_{jt}, np'_{jt})$. We denote by $\hat{m}_{N_s}(c; \boldsymbol{\kappa}, \boldsymbol{\varphi})$ a simulator of $m(c; \boldsymbol{\kappa}, \boldsymbol{\varphi})$ evaluated at the estimated policy functions, where N_s is the number of simulations. The inequality estimator minimizes $\min_{\boldsymbol{\varphi}} \mathcal{J}_{N_I} = \frac{1}{N_I} \sum_{k=1}^{N_I} \mathbf{1}\{\hat{m}_{N_s}(c; \boldsymbol{\kappa}, \boldsymbol{\varphi}) < 0\} \hat{m}_{N_s}(c; \boldsymbol{\kappa}, \boldsymbol{\varphi})^2$, where N_I is the number of inequalities.

4 Results

First, we discuss estimates of the multiproduct sales-generating function and the evolution of revenue productivity and demand shocks. We then focus on the role of entry regulations and discuss the determinants of the number of product categories and product-category sales competition in a store, followed by adjustment costs of variety, long-run profits and store benefits of adding products in different market types.

Sales-generating function estimates. Table 4 shows the estimates of the multiproduct sales-generating function in equation (5) by the OLS estimator and the nonparametric two-step estimator presented in Section 3.2.[54] The estimated coefficients of labor and inventories decrease from 0.786 (OLS) to 0.571 and from 1.037 (OLS) to 0.411, respectively, using the two-step estimator. The coefficient of capital increases, i.e., it is 0.061 (OLS) and 0.289 (the two-step estimator). These changes in the estimated coefficients are in line with the production function literature following Olley and Pakes (1996), which suggests an upper bias for the coefficients of labor and inventories when omitting to control for the correlation between inputs and productivity.

The estimates are consistent with the profit maximization behavior of multiproduct firms/stores because sales of a product category decrease when sales of other product categories increase (Mundlak, 1964). On average, a one percent increase in sales of other products decreases sales of a product category by 0.865 percent, suggesting relatively fierce competition for sales space in a store. The magnitude of the coefficient of the other product categories (α_y) is key for the productivity measure as it influences the input coefficients (labor, capital, inventories). The estimated elasticity of demand for product substitution is 3.480, which is in line with previous literature.

Stores in large and densely populated markets sell more per product category. The number of product categories and income have a positive impact on consumers' utility function and, therefore, on stores' market share. That consumers benefit from more product categories is consistent with previous literature, i.e., love-for-variety. On average, a store with a 30 percent market share gains 5 percent market share by adding one more product category.

Entry regulations and store primitives. Table 5 shows the estimates of the processes for productivity ω_{jt} and demand shocks μ_{jt}, i.e., equations (14) and (15). We reject the null hypothesis that the coefficients of demand shocks μ_{jt} in the productivity process equal zero (*p-value*=0.000). More liberal entry regulation has a positive impact on productivity, i.e., one more approval increases productivity by on average

[54]The two-step estimated coefficients are adjusted for the elasticity of substitution σ and the coefficient of other product categories $\tilde{\alpha}_y$ to allow for comparisons across specifications. The two-step estimator controls for the endogeneity of store input choices and entry regulation, and allows to identify two shocks separately.

0.120 percent.[55] However, the impact of entry regulations on productivity is decreasing in productivity and demand shocks. This implies high heterogeneity in stores' future productivity due to changes in regulation, which affects the long-run profits.

Demand shocks also have a positive impact on future revenue productivity, and the impact is increasing in productivity. A one percent increase in μ_{jt} raises productivity by on average 0.018 percent (see also Maican and Orth, 2021). We expect stores to learn from demand to improve future productivity in services where demand shocks affect inventory management, input choices and product variety that lead to productivity advances.

A key factor that drives the dynamics in productivity and demand shocks is persistence. The average persistence of the productivity process (0.869) is lower than the persistence of the store's demand shocks (0.943) (Table 5). The size of the persistence in productivity is similar to previous literature.[56]

Figure 3 presents box plots of the empirical distributions of revenue productivity and demand shocks for stores in different product-category quartiles in restrictive and liberal markets. First, the median revenue productivity ω_{jt} and demand shocks μ_{jt} are higher in liberal markets than in restrictive markets. Second, stores with higher productivity offer more product categories. Third, the interquartile range of demand shocks is lower in liberal than in restrictive markets for stores below the 75th percentile of the product category. Taken together, there is substantial heterogeneity in store-level primitives across stores and market types.

Entry regulations and product variety. Table 6 shows reduced-form evidence of the effect of productivity, demand shocks, investment and capital on the number of product categories and product-category competition for store sale space (product Herfindahl-Hirschman Index HHI) in restrictive and liberal markets.[57]

An increase in productivity intensifies competition for product space inside a store (lower HHI). The effect of productivity on product-category competition is decreasing in productivity and demand shocks in restrictive markets, whereas the reverse holds in liberal markets. Magnitudes are larger in liberal than restrictive markets. Stores with high demand shocks μ_{jt} have less intense competition between product categories (higher HHI) in both liberal and restrictive markets. However, the impact of demand shocks is increasing (decreasing) in productivity in restrictive (liberal) markets. This implies

[55] The average is computed based on the observed population density, where the largest marginal effect is approximately 9 percent (the standard deviation is 0.943). Based on an earlier study period (1996-2002) and no information on products and inventories, Maican and Orth (2015) also find a positive effect of more liberal entry regulations in different retail industries (with marginal effects up to 10 percent).

[56] See, e.g., Doraszelski and Jaumandreu (2013), Maican and Orth (2017).

[57] The OLS estimator is used for the HHI specification, and a quasi-Poisson estimator is used for the number of product categories. Both specifications include additional store controls and fixed effects for the local market, five-digit industries, and year. Sales at the product category are used to compute HHI inside store.

that product-category competition is less fierce if stores with high demand shocks in restrictive markets have high productivity.

Productivity gains raise product categories with larger magnitudes in liberal than restrictive markets. Stores in restrictive markets thus require larger productivity gains to obtain the same product-category increase as in liberal markets. Stores with high demand shocks offer fewer product categories, suggesting that stores reallocate resources from providing variety to decrease purchasing costs (e.g., providing shopping quality), in line with Bronnenberg (2015).

Table 7 presents the determinants of unique product categories in local markets. In other words, we aggregate the store-level information to the market level and analyze what drives product variety across markets. Markets with high median productivity have a larger unique number of product categories, with magnitudes being larger in liberal than in restrictive markets. Moreover, markets with high demand shocks have fewer product categories.

Our findings suggest substantial heterogeneity in the impact of productivity and demand shocks in restrictive and liberal markets. This heterogeneity enables us to understand drivers behind differences in variety across market types and is useful for designing policies that equate regional discrepancies.

Dynamic model estimates. The dynamic model is estimated using the indirect inference estimator. The value function is approximated using radial basis function networks, which provide a robust approximation of complex functions and their derivatives, such as the value function for a large state space with continuous variables (Mai-Duy and Tran-Cong, 2003; Sutton and Barto, 2018). We have to find the two optimal actions np_{jt} and a_{jt} by solving the dynamic model. The vector with the store's optimal actions is a solution of the system of equations constructed from the first-order conditions, which include the value function derivatives.

The results in Table 8 (Panel A) show that more liberal regulation decreases the adjustment costs of product categories. The decrease is larger for stores with many product categories (coefficients of the terms $exp(a_{jt}) \times r_{mt}$ and $np_{jt} \times r_{mt}$ are negative), reflecting that stores with many product categories benefit more from the marginal cost reduction following more liberal regulation. The coefficient of the term np_{jt}^2 is positive, implying decreasing returns to scale in the number of product categories in line with previous literature (e.g., Draganska and Jain, 2005). Last, stores with high demand for inventory before sales have higher marginal product-category adjustment costs. These findings speak to the fact that stores tradeoff the marginal adjustment cost of product categories with the long-run benefits.

The estimated dynamic model accurately predicts the number of product categories and inventory (Table 8, Panel B). This is because we allow for high heterogeneity in the adjustment cost of product categories. The mean of the value function (the long-run profits) is 232.6 M SEK. The median of the ratio between the long-run and short-run profits is 19.7 (Table 8, Panel C). In addition, the results show low errors in approximating the value function (median 3.6E-5), which ensures consistency of the estimation of the dynamic model.

Long-run profits and the benefits of variety. Table 9 shows incumbents' long-run profits, adjustment costs, and benefits of adding one more product category. We present results by policy-relevant market types for entry regulations and regional programs: rural, urban, restrictive and liberal markets. First, the median long-run profits in restrictive markets are approximately 38 percent higher than those in liberal markets, emphasizing that competition drives profitability differences between markets with contrasting regulations. The median long-run profits in urban markets are approximately 70 percent higher than those in rural markets. The difference in long-run profits between a store in the 90th percentile and the 10th percentile is over 250 M SEK in restrictive and rural markets, which is larger than in liberal and urban markets.

Second, the median adjustment cost of product categories is 29 percent higher in restrictive than in liberal markets. Urban markets have about 16 percent higher adjustment cost of product variety than rural markets. A store in the 90th percentile has 6.5-7 M SEK higher adjustment cost than a store in the 10th percentile. Restrictive markets have the largest dispersion in the adjustment costs related to offering product variety.

Third, by solving the store's dynamic optimization problem, we compute the increase in long-run profits from one more product category, i.e., the incumbent's long-run benefit from offering an additional product category for sale. Table 9 shows that the median benefit of increasing product categories is approximately 0.74 M SEK. The median benefit of adding variety is 1 percent lower in restrictive markets than in liberal markets. The median benefit of adding variety is 2 percent lower in rural rather than urban markets, reflecting less variety to consumers in rural areas. Stores located in restrictive markets have the highest dispersion in the long-run benefit of adding one more product category. For example, the long-run benefits for the store in the 90th percentile are approximately 1 M SEK higher than those in the 10th percentile. Variation in the benefit of adding variety across incumbents in different market types is a crucial component when we examine counterfactual regulatory designs and government subsidies.

5 Policy evaluation: More liberal entry regulation and cost subsidies

We use the estimated model to compute four counterfactual policy experiments. The first three counterfactuals increase competitive pressure from more liberal entry regulation: one additional PBL approval in all markets (CF_1), a 35 percent increase in the number of PBL approvals (CF_2), and doubling the number of PBL approvals (CF_3). Understanding the consequences of these regulatory regimes is highly relevant for policymakers who decide on PBL applications in local markets. The regulation policies use all channels through which entry regulations impact stores in our model, i.e., more liberal regulation reduces the adjustment cost of variety and improves future productivity that changes benefits from repositioning. Store's optimal repositioning balances changes in the marginal adjustment costs and changes in expected discounted future benefits from repositioning given the productivity improvements.

The fourth counterfactual (CF_4) evaluates a cost subsidy that yields zero marginal cost of adding product categories. This experiment mimics existing subsidies to incumbents in rural areas provided by the Swedish government (Section 2). We subsidize adjustment costs today without incentivizing stores to improve future productivity.[58] We contrast the subsidy in CF_4 with the generous liberalization of entry in CF_3 as they are cost equivalent at the industry level. For comparison, we also discuss two additional cost subsidy designs in Appendix E.

We compare store-level outcomes before and after hypothetical changes in entry regulations and cost subsidies in local markets. To compute the outcomes of a hypothetical change, we use the underlying primitives of the dynamic model and the estimated evolution of the state variables (i.e., productivity and demand shocks) to solve for the incumbents' number of product categories, sales per product category, and value function (i.e., long-run profits) using the Bellman equation. We report the average and standard deviation of changes in the incumbent's number of product categories (i.e., extensive margin), sales per product categories (i.e., intensive margin), store-level sales, inventory before sales, and the value function. We also report the share of stores that adjust their number of product categories and product-category entry and exit rates at the store

[58] The incumbents' short- and long-run profits are affected through changes in adjustment costs, which also impact sales because stores reposition in product categories and inventory. The implementation of an alternative policy changes the price equilibrium, which affects stores' optimal decisions (product variety and inventory). In our model, the new optimal equilibrium includes changes in price equilibrium due to a policy change. Without modeling regulation, the adjustment costs with product variety and solving store's dynamic optimization problem, Maican and Orth (2021) evaluate changes in product variety from investment subsidies on technology and mentoring support using a similar multiproduct sales-generating function.

level.[59] The results are presented for the four market types: rural, urban, restrictive and liberal markets. We particularly focus on markets with restrictive regulation and in rural locations, as a goal for policymakers is to equate conditions across geographic regions.[60]

More liberal entry regulation. The results from one additional PBL application in Table 10 (Panel A) show increasing repositioning in product categories among incumbents. Most repositioning occurs among incumbents in rural and restrictive markets (approximately 17 percent). Product-category entry rates are higher than exit rates in all markets. Product entry rates are on average 4 percent in restrictive markets, which is 2 percentage points higher than product exit rates. The difference between entry and exit rates is one percentage point in rural markets, where inventory before sales increases the most (6 percent). Product repositioning and new input choices, including inventory, increase the intensive product-category margin. The average increase in sales per product category is 6 percent in restrictive markets and 8 percent in rural markets. Stronger competitive pressure increases incumbents' future productivity and decreases product adjustment costs that, together with changes in product categories and inventory, result in higher long-run profits (value function). The average increase in the value function is 2-10 percent (10 percent in rural markets, 2 percent in urban markets, and 3-4 percent in restrictive and liberal markets). These findings suggest that incumbents benefit in the long run from one additional PBL application. Incumbents can learn the best practices from new entrants, which can encourage agglomeration economies and attract consumers to the local area.[61]

The findings from a 35 percent increase in approved PBL applications in CF_2 are consistent with those in CF_1 (Table 10, Panel B). Higher competitive pressure on incumbents in CF_2 than in CF_1 generates more entry and exit in product categories. Over 20 percent of all stores in restrictive markets adjust their product-category mix, which is 3 percentage points more than in CF_1. Product entry rates increase the most in markets with restrictive regulation (on average 5 percent). There is net entry of product categories and an average increase in incumbents' long-run profits in restrictive markets. In contrast, there is net exit of product categories and an average decrease in the incumbent's long-run profits in rural markets. These findings suggest that there is

[59] Although data limitations hinder us from directly computing consumer welfare, the number of product categories np_{jt} and part of demand shocks μ_{jt} associated with quality of shopping experience drive consumer surplus (Anderson et al., 1987; Anderson and De Palma, 2006). We also do not compute the change in total welfare, though total welfare gains come from changes in store surplus (profits) and consumer benefits of accessing a wider product variety that are provided by the dynamic model.

[60] See Section 2 for details. Rural and urban markets are defined based on the total population. Restrictive markets are those with below median PBL approvals per population density, and liberal otherwise.

[61] If we add uncertainty in demand shocks together with a more liberal regulation in CF_1, more stores adjust product-categories, entry rates increase whereas sales per product-category increase less.

room for more entry in restrictive markets. Consumers and incumbents in rural markets with limited demand are punished when competition increases substantially.

A doubling of the number of PBL approvals in CF_3 makes incumbents worse off (Table 10, Panel C). There is net entry of product categories in restrictive markets, whereas there is net exit in rural markets. Consumers in restrictive markets thus benefit from more products. Inventory adjusts to a larger extent, and incumbents keep more products in stock when competitive pressure is high. Generous liberalization decreases incumbents' long-run profits for all market types. Although such liberalization promotes productivity and decreases the adjustment costs of variety, it cannot compensate for the loss in future sales to new rivals. Intense competition thus reduces firm value. Incumbents in rural and restrictive markets are harmed the most under this policy design. For example, on average, the long-run profits decrease by 22 percent in rural markets and by 15 percent in restrictive markets. The reduction for incumbents in urban and liberal markets is approximately half.

Cost subsidy. The last experiment CF_4 that subsidizes the marginal adjustment cost of product categories sets the coefficient of the squared adjustment costs φ_2 in equation (17) to zero. This implies that the marginal adjustment cost difference between two stores is explained only by differences in inventory levels and the stringency of regulation. Table 10 (Panel D) shows that over 20 percent of stores adjust their product categories in rural and restrictive markets, which is approximately 3 percentage points more than in CF_3. The product entry rates are also substantially higher, on average approximately 6 percent in rural and restrictive markets. Rural incumbents gain long-run profits (by 24 percent, on average) as a result of a better product-category mix and lower adjustment costs. The same holds for the other market types, but the magnitudes are lower. Consumers are better off by accessing more variety, especially in rural and restrictive markets. This experiment shows that a reduction in 'diseconomies of scope' is relatively favorable for variety in markets where variety is sparser to begin with. When implementing such cost subsidies, however, the value of the reduction in differences in product variety across regions has to be weighted against the costs the government must pay, which can be high.

6 Conclusions

This paper assesses the impact of entry regulations on firms' incentives to adjust the product variety offered to consumers. An essential goal for policymakers is to ensure that consumers enjoy broad access to products and services regardless of where they live. The appropriate design of entry regulations and other policy tools to foster variety in local markets, such as subsidies, has been widely debated among policymakers and academics. However, remarkably little attention has been paid to the impact of entry regulations on retailers' repositioning of input resources and product variety.

We use a dynamic model of store adjustment in product categories and rich data to evaluate the long-run impact of different regulatory regimes in Swedish retail. The framework models economies of scale and scope in terms of offering multiple products and allows entry regulations to influence store productivity and the adjustment costs of product categories using information on stores' inputs and the local market environment. This research takes a first step towards understanding the role of entry regulations in shifting stores' incentives to offer product variety to consumers (i.e., more product categories), focusing on stores' reallocation of resources. We pay attention to rural and restrictively regulated markets that raise policy concerns to equate living conditions across geographic regions.

The empirical findings show that more liberal regulation decreases the adjustment cost of variety, increases productivity and spurs product-category repositioning. The median adjustment costs in product categories in restrictive markets is approximately 29 percent higher than those in liberal markets. Stores in restrictive markets have the largest dispersion in the long-run benefits of adding an additional product category. Stores in rural markets have the lowest benefit of adding variety, reflecting sparse variety in those markets.

Counterfactual policy experiments show that more liberal regulation of entry increases the number of product categories, especially in restrictive markets. A modest liberalization of entry increases incumbents' long-run profits as a result of productivity advances, lower adjustment costs and modified product-categories. The gains to consumers and incumbents are greatest in markets with restrictive regulation, implying a reduction in regional differences. A generous deregulation implies net exit of product categories in markets with limited demand and decreases long-run profits in all market types. A subsidy per product category for stores that utilize economies of scope induces high product-category entry rates. The policy is accurate from the point of view that variety increases relatively more in rural and restrictive markets than in urban and liberal markets. The cost for the government, which can be high for such policy, must be weighted against the value of equating differences across regions. Such subsidies are of interest to policymakers who want to equalize regional differences, as incumbents in rural and restrictive markets add relatively more product categories than incumbents in urban and liberal markets.

The dynamic framework with multiproduct technology can provide rich insights into changes in firms' behavior and the tradeoff between short-run costs and long-run benefits from various policies that aim to improve the product variety offered to consumers and ensure that the quality of life across regional areas is equitable.

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Table 1: Descriptive statistics

Year	Sales	Value added	Investment	No. of employees	Mean no. of product categories at store level	Mean of no. of PBL applications per population density	Corr. sales per product and no. of PBL applications per population density
2004	80.454	17.518	1.286	31,424	3.101	0.228	-0.020
2005	97.144	22.358	1.531	39,468	4.514	0.263	-0.005
2006	103.116	23.448	1.796	38,640	4.151	0.253	-0.004
2007	147.852	30.497	2.466	47,104	4.399	0.289	-0.020
2008	130.613	26.427	2.528	49,130	4.185	0.285	-0.040
2009	131.826	27.123	2.335	47,940	4.223	0.234	-0.019

NOTE: Sales (excl. VAT), value added, inventories (includes products bought), investment are measured in billions of 2000 SEK (1 USD= 7.3 SEK, 1 EUR= 9.3 SEK). Number of employees is measured in thousands. Sales per product category are computed at store level.

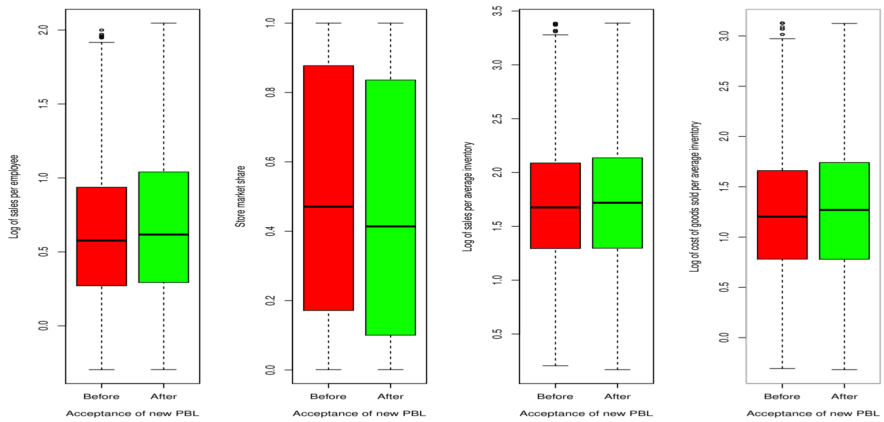

Figure 1: Store performance distributions before and after acceptance of new PBL applications

Table 2: The impact of accepted PBL applications on stores' product variety

	No. of products	Log of no. of products	Product sales entropy
	(1)	(2)	(3)
New applications accepted	0.344	0.047	-0.069
	(0.150)	(0.020)	(0.029)
Year fixed-effect	Yes	Yes	Yes
Subsector fixed-effect	Yes	Yes	Yes
Adjusted R2	0.236	0.204	0.242

NOTE: The independent variable is a dummy variable that takes the value one if there are new applications accepted in a local market. Clustered standard errors are in parentheses. Entropy measures store diversification in sales and is computed for each store j based on market share of each product category i inside store, i.e., $E_{jt} = \sum_i ms_{ijt}ln(ms_{ijt})$ (Bernard et al., 2011).

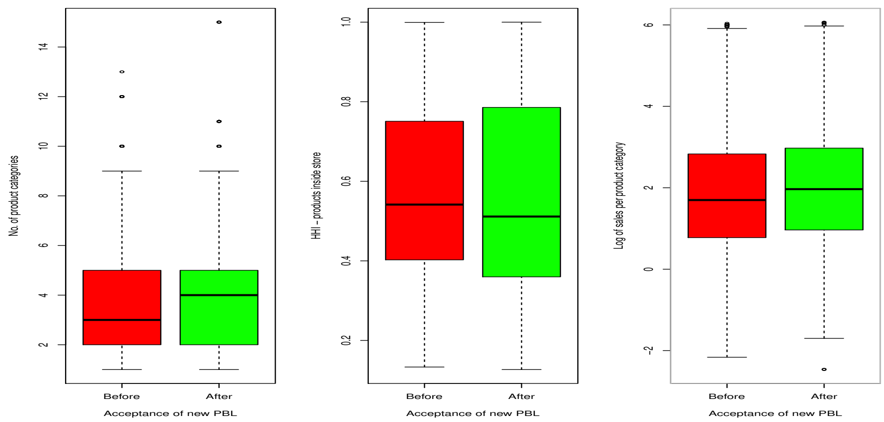

Figure 2: Multiproduct store's indicators before and after acceptance of new PBL applications

Table 3: The impact of entry regulation on the dynamics of stores' product variety

	Change in no. of products			Change in log of no. of products			Change in product sales entropy		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
No. of products in $t-1$		-0.396	-0.400						
		(0.053)	(0.014)						
Log of no. of products in $t-1$					-0.524	-0.526			
					(0.034)	(0.013)			
Product sales entropy in $t-1$								-0.372	-0.376
								(0.013)	(0.014)
Entry regulation in $t-1$	0.529	0.310	0.688	0.091	0.047	0.101	-0.071	-0.052	-0.170
	(0.241)	(0.174)	(0.289)	(0.040)	(0.022)	(0.051)	(0.034)	(0.030)	(0.053)
Year fixed-effect	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Market fixed-effect	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Subsector fixed-effect	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.193	0.381		0.132	0.455		0.119	0.305	
F-test (weak IV)			237.698			218.022			265.208
Sargan test (p-value)			0.230			0.094			0.961

NOTE: Clustered standard errors are in parentheses. Entropy measures store diversification in sales and is computed for each store j based on market share of each product category i inside store, i.e., $E_{jt} = \sum_i ms_{ijt} ln(ms_{ijt})$ (Bernard et al., 2011). The IV regressions use three instruments, i.e., the share of non-socialist seats in the local market, the number of approved applications in the neighboring municipalities, and one internal instrument exogenous variable (e.g., income and income squared) (see Lewbel, 2012).

Table 4: Estimation of multiproduct sales-generating function

	OLS		Two-step estimation	
	Estimate	Std.	Estimate	Std.
Log no. of employees	0.784	0.035	0.571	0.033
Log of capital	0.061	0.029	0.289	0.036
Log of inventory	1.037	0.021	0.411	0.054
Log of sales of other products	-0.896	0.009	-0.857	0.061
Log of sales outside option	-0.005	0.006	0.287	0.043
Log of population	0.014	0.022	0.176	0.032
Log of pop. density	0.018	0.016	0.697	0.032
Coef. of no. of products (ρ_{np})			0.213	0.096
Log of income	38.120	13.360	0.289	0.058
Log of income squared	-3.620	1.257	0.043	0.058
Elasticity of substitution			3.480	
Year fixed-effect	Yes		Yes	
Subsector fixed-effect	Yes		Yes	
R-squared	0.558			
No. of obs.	16,759		16,759	

NOTE: The dependent variable is the log of sales of a product category at the store level. Labor is measured as the number of full-time adjusted employees. Sales of other product categories are measured at the store level. Sales of outside option measures total sales of the other products of all other five-digit SNI codes at the local market. OLS regression controls for the current impact of entry regulation. OLS refers to ordinary least squares regression. Two-step estimation refers to the estimation method presented in Section 3. Reported standard errors (in parentheses) are computed using Ackerberg et al. (2012).

Table 5: Estimation of structural parameters: Productivity and demand shock processes

Productivity (ω_t) process			Demand shocks (μ_t) process		
	Estimate	Std.		Estimate	Std.
Productivity (ω_{t-1})	0.846	0.013	Demand shock (μ_{t-1})	0.987	0.018
Productivity squared (ω_{t-1}^2)	0.025	0.006	Demand shock squared (μ_{t-1}^2)	-0.012	0.004
Productivity cubic (ω_{t-1}^3)	-0.002	0.001	Demand shock cubic (μ_{t-1}^3)	-0.0006	0.0002
Demand shock (μ_{t-1})	0.025	0.004			
Prod.*Demand. shock ($\omega_{t-1} \times \mu_{t-1}$)	0.011	0.002			
Entry regulation (r_{t-1})	0.122	0.036			
Prod.*Entry reg. ($\omega_{t-1} \times r_{t-1}$)	-0.026	0.011			
Dem. sh.*Entry reg. ($\mu_{t-1} \times r_{t-1}$)	-0.028	0.006			
Year fixed-effects	Yes		Year fixed-effects	Yes	
Sub-sector fixed-effects	Yes		Year fixed-effects	Yes	
Adjusted R-squared	0.873		Adjusted R-squared	0.686	
Coefficients of ω_{t-1} terms are zero	F-test	p-value			
	1749.183	0.000			
Coefficients of μ_{t-1} terms are zero	F-test	p-value			
	23.601	0.000			
Coefficients of r_{t-1} terms are zero	F-test	p-value			
	7.599	0.000			
Persistence ($d\omega_t/d\omega_{t-1}$)	0.869		Persistence ($d\mu_t/d\mu_{t-1}$)	0.943	
Effect of demand shock ($d\omega_t/d\mu_{t-1}$)	0.025				
Effect of entry regulation ($d\omega_t/dr_{t-1}$)	0.077				

NOTE: Productivity is estimated using the two-step estimation method in Section 3. The mean values are presented for the marginal effects.

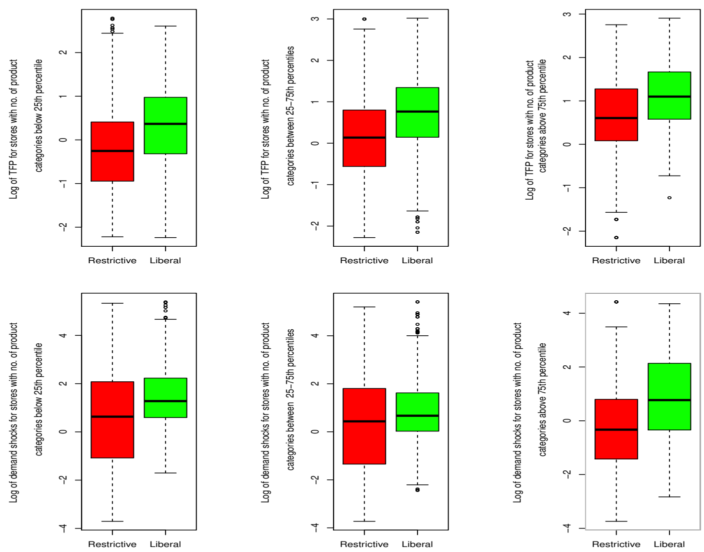

Figure 3: The relationship between the number of product categories, productivity, and demand shocks

Table 6: Determinants of product categories at the store level

	HHI product categories				No. of product categories			
	Restrictive		Liberal		Restrictive		Liberal	
	Est.	Std.	Est.	Std.	Est.	Std.	Est.	Std.
Productivity (ω_t)	-0.1131	0.0225	-0.1870	0.0429	0.3810	0.0468	0.6226	0.0693
Productivity squared (ω_t^2)	0.0039	0.0040	0.0125	0.0102	-0.0406	0.0148	-0.0616	0.0136
Demand shocks (μ_t)	0.1024	0.0127	0.1098	0.0223	-0.3574	0.0236	-0.3993	0.0390
Demand shocks squared (μ_t^2)	-0.0041	0.0005	-0.0057	0.0011	0.0165	0.0016	0.0208	0.0020
Prod.$\times$ Demand sh. ($\omega_t \times \mu_t$)	0.0081	0.0025	-0.0020	0.0032	-0.0064	0.0094	0.0087	0.0046
Log of capital stock (k_{t-1})	-0.0404	0.0089	-0.0494	0.0151	0.0926	0.0263	0.1259	0.0294
Log of investments (i_{t-1})	-0.0043	0.0052	0.0075	0.0058	0.0388	0.0109	0.0052	0.0135
Other store/market controls	Yes		Yes		Yes		Yes	
Sector fixed-effects	Yes		Yes		Yes		Yes	
Year fixed-effects	Yes		Yes		Yes		Yes	
Market fixed-effects	Yes		Yes		Yes		Yes	
Adj. R^2	0.5404		0.6040					

NOTE: All regressions include an intercept. OLS estimator is used for HHI regressions, where the dependent variable, i.e HHI, is computed based on sales product categories. Quasi-Poisson estimator is used for the number of product categories regressions. Additional store and market controls include: inventories, wages, population, population density, income. Standard errors are clustered at sector level.

Table 7: Determinants of the number of unique product categories in local markets

	Restrictive markets		Liberal markets	
	Est.	Std.	Est.	Std.
Productivity (ω_t)	0.2212	0.0447	0.3008	0.0378
Productivity squared (ω_t^2)	-0.0164	0.0097	-0.0167	0.0129
Demand shocks (μ_t)	-0.2298	0.0288	-0.2827	0.0323
Demand shocks squared (μ_t^2)	0.0114	0.0018	0.0149	0.0018
Log of capital stock (k_{t-1})	0.0809	0.0164	0.1110	0.0203
Log of investments (i_{t-1})	0.0386	0.0127	0.0060	0.0107
Sector fixed-effects	Yes		Yes	
Year fixed-effects	Yes		Yes	
Market fixed-effects	Yes		Yes	
Adj. R^2	0.5554		0.4024	

NOTE: Dependent variable is the log of unique number of product categories at local market and sector level. Controls include median values at the local market, sector and year level. OLS estimator is used. All regressions include an intercept, and additional market controls (median values), i.e., population, population density, and income. Standard errors are clustered at market level.

Table 8: Estimation of dynamic parameters

Panel A: Estimation of adjustment cost in product variety		
	Estimate	Std.
No. of product categories (np_{jt})	0.1109	0.0347
No. of product categories squared (np_{jt}^2)	0.1018	0.0095
Inventory before sales squared($exp(a_{jt})^2$)	0.0017	0.0006
No. of product categ. $\times$ Inv. before sales ($np_{jt} \times exp(a_{jt})$)	0.0993	0.0461
No. of product categ. $\times$ Regulation ($np_{jt} \times r_{mt}$)	-0.1029	0.0378
Inv. before sales $\times$ Regulation ($exp(a_{jt}) \times r_{mt}$)	-0.3119	0.0946
Panel B: Model prediction		
	Observed	Predicted
No. product categories		
Mean	3.8748	3.9963
Std.	1.7494	1.6907
Log of inventory before sales		
Mean	2.2143	2.0001
Std.	0.9656	1.1520
Panel C: Value function approximation		
Value function		
Mean	232.5870	
Std.	301.8850	
Median value function over net profits	19.6521	
Median approximation error	-2.3420E-5	

NOTE: Standard errors are computed using subsampling.

Table 9: Stores' long-run profits and the benefits of increasing product variety by market type

	Type of market							
	Rural		Urban		Restrictive		Liberal	
	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR
Value function	121.7120	262.4670	206.9500	230.7650	206.1960	250.0120	181.6170	239.2370
Adjustment cost	2.5182	6.9690	2.9221	7.3515	3.1343	7.6116	2.4381	6.5205
Marginal benef. of variety	0.7498	0.8949	0.7353	0.9402	0.7333	0.9452	0.7376	0.9046

NOTE: Figures are in thousand SEK. $IQR = Q_{90} - Q_{10}$. Q_{10}, Q_{50}, and Q_{90} are 10th, median, and 90th percentile.

Table 10: Counterfactual experiments: More liberal entry regulation and cost subsidies

	Type of market							
	Rural		Urban		Restrictive		Liberal	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std
Panel A: CF_1 - One more approved PBL application in all markets								
Share of stores with product adjust.	0.1677		0.1543		0.1692		0.1444	
Product category entry rate	0.0287	0.1160	0.0336	0.1759	0.0417	0.2145	0.0238	0.0983
Product category exit rate	0.0243	0.0924	0.0211	0.0813	0.0235	0.0858	0.0198	0.0809
Inventory before sales	0.0607	0.2866	0.0127	0.1535	0.0090	0.1788	0.0334	0.1905
Sales	0.0212	0.1336	0.0114	0.1872	0.0094	0.2094	0.0168	0.1422
Sales per product	0.0892	0.9298	0.0507	0.7504	0.0644	0.9253	0.0508	0.6158
Adjustment cost	-0.0300	0.2161	0.0008	0.2437	0.0123	0.2527	-0.0215	0.2241
Value function	0.1008	0.7740	0.0217	0.1273	0.0319	0.4178	0.0396	0.2586
Panel B: CF_2 - An increase in approved PBL applications by 35 percent								
Share of stores with product adjust.	0.1538		0.1857		0.2097		0.1510	
Product category entry rate	0.0245	0.1295	0.0394	0.1746	0.0518	0.2156	0.0220	0.0980
Product category exit rate	0.0274	0.0977	0.0291	0.1030	0.0332	0.1122	0.0245	0.0909
Inventory before sales	0.0654	0.2110	0.0510	0.1867	0.0294	0.2003	0.0774	0.1784
Sales	0.0270	0.1195	0.0436	0.2555	0.0327	0.2749	0.0487	0.1941
Sales per product	0.1095	0.9708	0.1305	1.0543	0.1450	1.2389	0.1090	0.7962
Adjustment cost	-0.0329	0.2280	-0.0125	0.3203	0.0382	0.3315	-0.0696	0.2692
Value function	-0.0372	0.3818	0.0876	0.3646	0.0610	0.3793	0.0713	0.3618
Panel C: CF_3 - Double the number of approved PBL applications								
Share of stores with product adjust.	0.1825		0.1965		0.2068		0.1816	
Product category entry rate	0.0165	0.0691	0.0350	0.1818	0.0434	0.2178	0.0205	0.0960
Product category exit rate	0.0307	0.0964	0.0315	0.0989	0.0346	0.1100	0.0281	0.0854
Inventory before sales	0.1798	0.4441	0.1113	0.2602	0.0379	0.1819	0.2069	0.3632
Sales	0.0509	0.1371	0.0622	0.2209	0.0323	0.2662	0.0881	0.1243
Sales per product	0.0836	0.2071	0.1366	0.9402	0.1435	1.2089	0.1121	0.1782
Adjustment cost	-0.1103	0.2695	-0.0855	0.3012	0.0025	0.2697	-0.1812	0.2932
Value function	-0.2208	0.6138	-0.0920	0.3350	-0.1521	0.4093	-0.0749	0.3822
Panel D: CF_4 - Cost subsidy per product-category that varies with the number of product categories								
Share of stores with product adjust.	0.2048		0.2069		0.2321		0.1811	
Product category entry rate	0.0598	0.2078	0.0472	0.1712	0.0627	0.2100	0.0361	0.1382
Product category exit rate	0.0147	0.0629	0.0222	0.089	0.0234	0.0928	0.0184	0.0766
Inventory before sales	0.0038	0.1322	0.0066	0.1384	0.0123	0.1676	-0.0002	0.0978
Sales	0.0137	0.0792	0.0177	0.1934	0.0214	0.1986	0.0127	0.1564
Sales per product	-0.0101	0.1218	0.0602	0.842	0.0655	0.9409	0.0303	0.5405
Adjustment cost	-0.3438	0.273	-0.2825	0.5536	-0.2765	0.278	-0.3099	0.6746
Value function	0.2446	0.9803	0.0185	0.4633	0.0569	0.6164	0.0593	0.5694

NOTE: Figures represent growth changes. The counterfactuals CF_3 and CF_4 are cost equivalent at the industry level.

Online Appendix: Entry Regulations and Product Variety in Retail

Florin Maican and Matilda Orth[1]

Appendix A: General properties of the multiproduct service function

To simplify the notation, we omit the index of the firm and period and denote the group the store service inputs into the vector $\mathbf{V}$. For example, in our empirical implementation $\mathbf{V} = (L, K, A)$. We consider the general service generating function, i.e.,

$$F(\mathbf{Q}, \mathbf{V}) = G(\mathbf{Q}) - H(\mathbf{V}) = 0 \tag{1-a}$$

where $G(\mathbf{Q}) = Q_1^{\tilde{\alpha}_1} \times \cdots \times Q_{np}^{\tilde{\alpha}_{np}} exp(\tilde{\gamma}_1 Q_1 + \cdots + \tilde{\gamma}_{np} Q_{np})$; $H(\mathbf{V}) = V_1^{\tilde{\beta}_1} \times \cdots \times V_m^{\tilde{\beta}_m} exp(\tilde{\omega})$; $\mathbf{Q}$ is the vector of service output; Q_i is the i-th service output of the store, $(i = \overline{1, np})$; and V_j is the j-th service input of the store, $(j = \overline{1, m})$. In what follows, we use the i to index the service outputs and j to index the inputs.

Assuming that the prices are given, the Lagrangean function of the profit maximization at the store level is given by

$$\max_{\mathbf{V}} \mathcal{L} = \mathbf{P}'\mathbf{Q} - \mathbf{W}'\mathbf{V} - \lambda F(\mathbf{Q}, \mathbf{V}), \tag{2-a}$$

where $\mathbf{P}$ and $\mathbf{W}$ are the vectors of output and input prices, respectively. The first-order conditions (FOC) under competition are

$$\begin{aligned} &P_i - \lambda F_i = 0, \quad i = \overline{1, np} \\ &W_j + \lambda F_j = 0, \quad j = \overline{1, m} \end{aligned} \tag{3-a}$$

where $F_i = \partial F / \partial Q_i$ and $F_j = \partial F / \partial V_j$. The FOC (3-a) conditions imply that $sign(\lambda) = sign(F_i)$ and $sign(\lambda) = -sign(F_j)$. The derivatives of the implicit function with respect

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to inputs and outputs, i.e. F_i and F_j are

$$\begin{aligned} F_i &= G(\mathbf{Q})\left(\frac{\tilde{\alpha}_i}{Q_i} + \tilde{\gamma}_i\right), \quad i = \overline{1, np} \\ F_j &= -H(\mathbf{V})\frac{\tilde{\beta}_j}{V_j}, \quad j = \overline{1, m} \end{aligned} \tag{4-a}$$

The cross derivatives of the Lagrangean are the following: $\partial^2 \mathcal{L}/\partial^2 \lambda = 0$; $\partial^2 \mathcal{L}/\partial \lambda \partial Q_i = -F_i$; $\partial^2 \mathcal{L}/\partial \lambda \partial V_j = -F_j$; $\partial^2 \mathcal{L}/\partial Q_i \partial Q_{i'} = -\lambda F_{ii'}$; $\partial^2 \mathcal{L}/\partial V_j \partial V_{j'} = -\lambda F_{jj'}$; and $\partial^2 \mathcal{L}/\partial Q_i \partial V_j = -\lambda F_{ij}$. The determinant of the bordered Hessian matrix $D_{\mathcal{L}}$ is given by

$$D_{\mathcal{L}} = \begin{vmatrix} \frac{\partial^2 L}{\partial \lambda \partial \lambda} & \frac{\partial^2 L}{\partial \lambda \partial Q_i} & \frac{\partial^2 L}{\partial \lambda \partial V_j} \\ \frac{\partial^2 L}{\partial Q_i \partial \lambda} & \frac{\partial^2 L}{\partial Q_i \partial Q_i} & \frac{\partial^2 L}{\partial Q_i \partial V_j} \\ \frac{\partial^2 L}{\partial V_j \partial \lambda} & \frac{\partial^2 L}{\partial V_j \partial Q_i} & \frac{\partial^2 L}{\partial V_j \partial V_{j'}} \end{vmatrix} = \begin{vmatrix} 0 & -\mathbf{F}_i & -\mathbf{F}_j \\ -\mathbf{F}_i & -\lambda \mathbf{F}_{ii'} & -\lambda \mathbf{F}_{ij} \\ -\mathbf{F}_j & -\lambda \mathbf{F}_{ji'} & -\lambda \mathbf{F}_{jj'} \end{vmatrix}, \tag{5-a}$$

where the cross derivatives of elements of the block matrices of the determinant of the hessian matrix are the following

$$\begin{aligned} &\text{Product-product: } F_{ii} = \frac{F_i^2}{G(\mathbf{Q})} - G(\mathbf{Q})\frac{\tilde{\alpha}_i}{Q_i^2}, \quad i = \overline{1, np} \\ &\text{Product-product: } F_{ii'} = \frac{F_i F_{i'}}{G(\mathbf{Q})}, \quad i \neq i' \quad i, i' = \overline{1, np} \\ &\text{Input-input: } F_{jj} = -\frac{F_j^2}{H(\mathbf{V})} + H(\mathbf{V})\frac{\tilde{\beta}_j}{V_j^2}, \quad j = \overline{1, m} \\ &\text{Input-input: } F_{jj'} = -\frac{F_j F_{j'}}{H(\mathbf{V})}, \quad j \neq j' \quad j, j' = \overline{1, m} \\ &\text{Product-input: } F_{ij} = 0, \quad i = \overline{1, np}, \quad j = \overline{1, m}. \end{aligned} \tag{6-a}$$

The second-order condition of the profit maximization requires the sign of the determinant of the bordered Hessian matrix $D_{\mathcal{L}}$ is $(-1)^{np+m}$. To proof this, we rewrite the determinant $D_{\mathcal{L}}$ as

$$D_{\mathcal{L}} = \begin{vmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{vmatrix}, \tag{7-a}$$

where $\mathbf{A} = 0$ (1×1 matrix); $\mathbf{B} = [-\mathbf{F}_i, -\mathbf{F}_j]^T$ ($1 \times (np + m)$); $\mathbf{C} = [-\mathbf{F}_i, -\mathbf{F}_j]$ ($(np + m) \times 1$); and

$$\mathbf{D} = \begin{bmatrix} -\lambda \mathbf{F}_{ii'} & \mathbf{0} \\ \mathbf{0} & -\lambda \mathbf{F}_{jj'} \end{bmatrix}.$$

Using Schur complement decomposition, we have that

$$D_{\mathcal{L}} = det(\mathbf{D}) det(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}). \tag{8-a}$$

Because the matrix $\mathbf{D}$ is diagonal, its inverse is given by

$$\mathbf{D}^{-1} = (-\lambda)^{-1} \begin{bmatrix} \mathbf{F}_{ii'}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{jj'}^{-1} \end{bmatrix}, \tag{9-a}$$

and the determinant of $\mathbf{D}$ is

$$det(\mathbf{D}) = (-1)^{-(np+m)}(\lambda)^{-(np+m)}det(\mathbf{F}_{ii'})det(\mathbf{F}_{jj'}). \tag{10-a}$$

The product $\mathbf{BD}^{-1}\mathbf{C}$ can be rewritten as

$$\mathbf{BD}^{-1}\mathbf{C} = -\lambda^{-1}[\mathbf{F}_i^T\mathbf{F}_{ii'}^{-1}\mathbf{F}_i + \mathbf{F}_j^T\mathbf{F}_{jj'}^{-1}\mathbf{F}_j]. \tag{11-a}$$

Therefore,

$$det(A - \mathbf{BD}^{-1}\mathbf{C}) = (\lambda)^{-1}[\mathbf{F}_i^T\mathbf{F}_{ii'}^{-1}\mathbf{F}_i + \mathbf{F}_j^T\mathbf{F}_{jj'}^{-1}\mathbf{F}_j]. \tag{12-a}$$

Thus, the determinant of the bordered Hessian matrix is given by

$$D_{\mathfrak{L}} = (-1)^{-(np+m)}(\lambda)^{-(np+m+1)}det(\mathbf{F}_{ii'})det(\mathbf{F}_{jj'})[\mathbf{F}_i^T\mathbf{F}_{ii'}^{-1}\mathbf{F}_i + \mathbf{F}_j^T\mathbf{F}_{jj'}^{-1}\mathbf{F}_j]. \tag{13-a}$$

The block matrices $\mathbf{F}_{ii'}$ and $\mathbf{F}_{jj'}$ have important properties that can be used to compute their inverse and the determinant. The matrices $\mathbf{F}_{ii'}$ and $\mathbf{F}_{jj'}$ can be written as

$$\mathbf{F}_{ii'} = \begin{bmatrix} \frac{F_1F_1}{G(\mathbf{Q})} & \cdots & \frac{F_1F_{np}}{G(\mathbf{Q})} \\ \vdots & \ddots & \vdots \\ \frac{F_{np}F_1}{G(\mathbf{Q})} & \cdots & \frac{F_{np}F_{np}}{G(\mathbf{Q})} \end{bmatrix} + \begin{bmatrix} -\frac{\tilde{\alpha}_1}{Q_1^2}G(\mathbf{Q}) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & -\frac{\tilde{\alpha}_{np}}{Q_{np}^2}G(\mathbf{Q}) \end{bmatrix}$$

and

$$\mathbf{F}_{jj'} = \begin{bmatrix} -\frac{F_1F_1}{H(\mathbf{V})} & \cdots & -\frac{F_1F_m}{H(\mathbf{V})} \\ \vdots & \ddots & \vdots \\ -\frac{F_mF_1}{H(\mathbf{V})} & \cdots & -\frac{F_mF_{np}}{H(\mathbf{V})} \end{bmatrix} + \begin{bmatrix} \frac{\tilde{\beta}_1}{V_1^2}H(\mathbf{V}) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \frac{\tilde{\beta}_m}{V_m^2}H(\mathbf{V}) \end{bmatrix}.$$

We introduce new notations for the vectors of derivatives in outputs and inputs, i.e.,

$$\mathbf{u}_i^T = \left[\frac{-F_1}{G(\mathbf{Q})^{\frac{1}{2}}}, \cdots, \frac{-F_{np}}{G(\mathbf{Q})^{\frac{1}{2}}}\right]$$

$$\mathbf{u}_j^T = \left[\frac{-F_1}{H(\mathbf{V})^{\frac{1}{2}}}, \cdots, \frac{-F_m}{H(\mathbf{V})^{\frac{1}{2}}}\right]$$

$$\mathbf{v}_j^T = \left[\frac{F_1}{H(\mathbf{V})^{\frac{1}{2}}}, \cdots, \frac{F_m}{H(\mathbf{V})^{\frac{1}{2}}}\right]$$

$$\tilde{\mathbf{F}}_{ii'} = \begin{bmatrix} -\frac{\tilde{\alpha}_1}{Q_1^2}G(\mathbf{Q}) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & -\frac{\tilde{\alpha}_{np}}{Q_{np}^2}G(\mathbf{Q}) \end{bmatrix}$$

$$\tilde{\mathbf{F}}_{jj'} = \begin{bmatrix} \frac{\tilde{\beta}_1}{V_1^2}H(\mathbf{V}) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \frac{\tilde{\beta}_m}{V_m^2}H(\mathbf{V}) \end{bmatrix}.$$

The cross-derivative matrices $\mathbf{F}_{ii'}$ and $\mathbf{F}_{jj'}$ can be decomposed as

$$\begin{aligned} \mathbf{F}_{ii'} &= \tilde{\mathbf{F}}_{ii'} + \mathbf{u}_i\mathbf{u}_i^T \\ \mathbf{F}_{jj'} &= \tilde{\mathbf{F}}_{jj'} + \mathbf{u}_j\mathbf{v}_j^T. \end{aligned} \tag{14-a}$$

Based on these decompositions, we can compute the inverses and the determinants of $\mathbf{F}_{ii'}$ and $\mathbf{F}_{jj'}$ using Sherman-Morrison formula, i.e.,

$$(\tilde{\mathbf{F}}_{ii'} + \mathbf{u}_i\mathbf{u}_i^T)^{-1} = \tilde{\mathbf{F}}_{ii'}^{-1} - \frac{\tilde{\mathbf{F}}_{ii'}^{-1}\mathbf{u}_i\mathbf{u}_i^T\tilde{\mathbf{F}}_{ii'}^{-1}}{1 + \mathbf{u}_i^T\tilde{\mathbf{F}}_{ii'}^{-1}\mathbf{u}_i} \tag{15-a}$$

$$(\tilde{\mathbf{F}}_{jj'} + \mathbf{u}_j\mathbf{v}_i^T)^{-1} = \tilde{\mathbf{F}}_{jj'}^{-1} - \frac{\tilde{\mathbf{F}}_{jj'}^{-1}\mathbf{u}_j\mathbf{v}_j^T\tilde{\mathbf{F}}_{jj'}^{-1}}{1 + \mathbf{v}_j^T\tilde{\mathbf{F}}_{jj'}^{-1}\mathbf{u}_j} \tag{16-a}$$

$$det(\tilde{\mathbf{F}}_{ii'} + \mathbf{u}_i\mathbf{u}_i^T) = (1 + \mathbf{u}_i^T\tilde{\mathbf{F}}_{ii'}^{-1}\mathbf{u}_i)det(\tilde{\mathbf{F}}_{ii'}) \tag{17-a}$$

$$det(\tilde{\mathbf{F}}_{jj'} + \mathbf{u}_j\mathbf{v}_j^T) = (1 + \mathbf{u}_j^T\tilde{\mathbf{F}}_{jj'}^{-1}\mathbf{u}_j)det(\tilde{\mathbf{F}}_{jj'}). \tag{18-a}$$

The inverse of the diagonal matrices $\tilde{\mathbf{F}}_{ii'}$ and $\tilde{\mathbf{F}}_{jj'}$ are given by

$$\tilde{\mathbf{F}}_{ii'}^{-1} = \begin{bmatrix} -\frac{Q_1^2}{\tilde{\alpha}_1 G(\mathbf{Q})} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & -\frac{Q_{np}^2}{\tilde{\alpha}_{np} G(\mathbf{Q})} \end{bmatrix} \tag{19-a}$$

$$\tilde{\mathbf{F}}_{jj'}^{-1} = \begin{bmatrix} \frac{V_1^2}{\tilde{\beta}_1 H(\mathbf{V})} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \frac{V_m^2}{\tilde{\beta}_m H(\mathbf{V})} \end{bmatrix}. \tag{20-a}$$

We use Sherman-Morrison formula to evaluate the terms $\mathbf{F}_i^T \mathbf{F}_{ii'}^{-1} \mathbf{F}_i$ and $\mathbf{F}_j^T \mathbf{F}_{jj'}^{-1} \mathbf{F}_j$, i.e.,

$$\begin{aligned} \mathbf{F}_i^T \mathbf{F}_{ii'}^{-1} \mathbf{F}_i = & \; G(\mathbf{Q})^{\frac{1}{2}} \mathbf{u}_i^T \left(\tilde{\mathbf{F}}_{ii'}^{-1} - \frac{\tilde{\mathbf{F}}_{ii'}^{-1} \mathbf{u}_i \mathbf{u}_i^T \tilde{\mathbf{F}}_{ii'}^{-1}}{1+\mathbf{u}_i^T \tilde{\mathbf{F}}_{ii'}^{-1} \mathbf{u}_i} \right) \mathbf{u}_i G(\mathbf{Q})^{\frac{1}{2}} \\ & = G(\mathbf{Q}) \frac{\mathbf{u}_i^T \tilde{\mathbf{F}}_{ii'}^{-1} \mathbf{u}_i}{1+\mathbf{u}_i^T \tilde{\mathbf{F}}_{ii'}^{-1} \mathbf{u}_i} \end{aligned} \tag{21-a}$$

$$\begin{aligned} \mathbf{F}_j^T \mathbf{F}_{jj'}^{-1} \mathbf{F}_j = & \; -H(\mathbf{V})^{\frac{1}{2}} \mathbf{v}_j^T \left(\tilde{\mathbf{F}}_{jj'}^{-1} - \frac{\tilde{\mathbf{F}}_{jj'}^{-1} \mathbf{u}_j \mathbf{v}_j^T \tilde{\mathbf{F}}_{jj'}^{-1}}{1+\mathbf{v}_j^T \tilde{\mathbf{F}}_{jj'}^{-1} \mathbf{u}_j} \right) \mathbf{u}_j H(\mathbf{V})^{\frac{1}{2}} \\ & = -H(\mathbf{V}) \frac{\mathbf{v}_j^T \tilde{\mathbf{F}}_{jj'}^{-1} \mathbf{u}_j}{1+\mathbf{v}_j^T \tilde{\mathbf{F}}_{jj'}^{-1} \mathbf{u}_j}. \end{aligned} \tag{22-a}$$

The terms $\mathbf{u}_i^T \tilde{\mathbf{F}}_{ii'}^{-1} \mathbf{u}_i$ and $\mathbf{v}_j^T \tilde{\mathbf{F}}_{jj'}^{-1} \mathbf{u}_j$ can be computed as follows:

$$\begin{aligned} \mathbf{u}_i^T \tilde{\mathbf{F}}_{ii'}^{-1} \mathbf{u}_i & = \left[\frac{-F_1}{G(\mathbf{Q})^{\frac{1}{2}}}, \cdots, \frac{-F_{np}}{G(\mathbf{Q})^{\frac{1}{2}}} \right] \begin{bmatrix} -\frac{Q_1^2}{\tilde{\alpha}_1 G(\mathbf{Q})} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & -\frac{Q_{np}^2}{\tilde{\alpha}_{np} G(\mathbf{Q})} \end{bmatrix} \begin{bmatrix} \frac{-F_1}{G(\mathbf{Q})^{\frac{1}{2}}} \\ \vdots \\ \frac{-F_{np}}{G(\mathbf{Q})^{\frac{1}{2}}} \end{bmatrix} \\ & = -\sum_{i=1}^{np} \frac{1}{\tilde{\alpha}_i} \frac{F_i^2 Q_i^2}{G(\mathbf{Q})^2} \end{aligned}$$

$$\begin{aligned} \mathbf{v}_j^T \tilde{\mathbf{F}}_{jj'}^{-1} \mathbf{u}_j & = \left[\frac{F_1}{H(\mathbf{V})^{\frac{1}{2}}}, \cdots, \frac{F_m}{H(\mathbf{V})^{\frac{1}{2}}} \right] \begin{bmatrix} \frac{V_1^2}{\tilde{\beta}_1 H(\mathbf{V})} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \frac{V_m^2}{\tilde{\beta}_{np} H(\mathbf{V})} \end{bmatrix} \begin{bmatrix} \frac{-F_1}{H(\mathbf{V})^{\frac{1}{2}}} \\ \vdots \\ \frac{-F_m}{H(\mathbf{V})^{\frac{1}{2}}} \end{bmatrix} \\ & = -\sum_{j=1}^{m} \frac{1}{\tilde{\beta}_j} \frac{F_j^2 V_j^2}{H(\mathbf{V})^2}. \end{aligned}$$

Therefore, we have

$$\begin{aligned}
\mathbf{F}_i^T\mathbf{F}_{ii'}^{-1}\mathbf{F}_i &= G(\mathbf{Q})\frac{-\sum_{i=1}^{np}\frac{1}{\tilde{\alpha}_i}\frac{F_i^2Q_i^2}{G(\mathbf{Q})^2}}{1-\sum_{i=1}^{np}\frac{1}{\tilde{\alpha}_i}\frac{F_i^2Q_i^2}{G(\mathbf{Q})^2}} \\
\mathbf{F}_j^T\mathbf{F}_{jj'}^{-1}\mathbf{F}_j &= H(\mathbf{V})\frac{-\sum_{j=1}^{m}\frac{1}{\tilde{\beta}_j}\frac{F_j^2V_j^2}{H(\mathbf{V})^2}}{1-\sum_{j=1}^{m}\frac{1}{\tilde{\beta}_j}\frac{F_j^2V_j^2}{H(\mathbf{V})^2}}
\end{aligned} \tag{23-a}$$

The next step is to compute the determinants of $\mathbf{F}_{ii'}$ and $\mathbf{F}_{jj'}$, i.e.,

$$\begin{aligned}
det(\mathbf{F}_{ii'}) &= (1+\mathbf{u}_i^T\tilde{\mathbf{F}}_{ii'}^{-1}\mathbf{u}_i)det(\tilde{\mathbf{F}}_{ii'}) \\
&= \left(1-\sum_{i=1}^{np}\frac{1}{\tilde{\alpha}_i}\frac{F_i^2Q_i^2}{G(\mathbf{Q})^2}\right)\prod_{i=1}^{np}\frac{-\tilde{\alpha}_i}{Q_i^2}G(\mathbf{Q})
\end{aligned} \tag{24-a}$$

$$\begin{aligned}
det(\mathbf{F}_{jj'}) &= (1+\mathbf{v}_j^T\tilde{\mathbf{F}}_{jj'}^{-1}\mathbf{u}_j)det(\tilde{\mathbf{F}}_{ii'}) \\
&= \left(1-\sum_{j=1}^{m}\frac{1}{\tilde{\beta}_j}\frac{F_j^2V_j^2}{H(\mathbf{V})^2}\right)\prod_{j=1}^{m}\frac{\tilde{\beta}_j}{V_j^2}H(\mathbf{V}).
\end{aligned} \tag{25-a}$$

Replacing expressions (10-a), (12-a), (23-a), (24-a), and (25-a) in (8-a), we have

$$\begin{aligned}
D_{\mathcal{L}} = \quad & \lambda(-\lambda)^{-(np+m+2)}\left(1-\sum_{i=1}^{np}\frac{1}{\tilde{\alpha}_i}\frac{F_i^2Q_i^2}{G(\mathbf{Q})^2}\right)\left(\prod_{i=1}^{np}\frac{-\tilde{\alpha}_i}{Q_i^2}G(\mathbf{Q})\right) \\
\times \quad & \left(1-\sum_{j=1}^{m}\frac{1}{\tilde{\beta}_j}\frac{F_j^2V_j^2}{H(\mathbf{V})^2}\right)\left(\prod_{j=1}^{m}\frac{\tilde{\beta}_j}{V_j^2}H(\mathbf{V})\right) \\
\times \quad & \left[G(\mathbf{Q})\frac{-\sum_{i=1}^{np}\frac{1}{\tilde{\alpha}_i}\frac{F_i^2Q_i^2}{G(\mathbf{Q})^2}}{1-\sum_{i=1}^{np}\frac{1}{\tilde{\alpha}_i}\frac{F_i^2Q_i^2}{G(\mathbf{Q})^2}} - H(\mathbf{V})\frac{-\sum_{j=1}^{m}\frac{1}{\tilde{\beta}_j}\frac{F_j^2V_j^2}{H(\mathbf{V})^2}}{1-\sum_{j=1}^{m}\frac{1}{\tilde{\beta}_j}\frac{F_j^2V_j^2}{H(\mathbf{V})^2}}\right],
\end{aligned} \tag{26-a}$$

where $\frac{F_i^2Q_i^2}{G(\mathbf{Q})^2} = (\tilde{\alpha}_i + \tilde{\gamma}_iQ_i)^2$ and $\frac{F_j^2V_j^2}{H(\mathbf{V})^2} = \tilde{\beta}_j^2$. We simplify the expression of $D_{\mathcal{L}}$ by introducing new notations for each term, i.e.,

$$\begin{aligned}
T_1 &= \left(1-\sum_{i=1}^{np}\frac{1}{\tilde{\alpha}_i}\frac{F_i^2Q_i^2}{G(\mathbf{Q})^2}\right) \\
T_2 &= \left(\prod_{i=1}^{np}\frac{-\tilde{\alpha}_i}{Q_i^2}G(\mathbf{Q})\right) \\
T_3 &= \left(1-\sum_{j=1}^{m}\frac{1}{\tilde{\beta}_j}\frac{F_j^2V_j^2}{H(\mathbf{V})^2}\right) \\
T_4 &= \left(\prod_{j=1}^{m}\frac{\tilde{\beta}_j}{V_j^2}H(\mathbf{V})\right) \\
T_5 &= \left[G(\mathbf{Q})\frac{-\sum_{i=1}^{np}\frac{1}{\tilde{\alpha}_i}\frac{F_i^2Q_i^2}{G(\mathbf{Q})^2}}{1-\sum_{i=1}^{np}\frac{1}{\tilde{\alpha}_i}\frac{F_i^2Q_i^2}{G(\mathbf{Q})^2}} - H(\mathbf{V})\frac{-\sum_{j=1}^{m}\frac{1}{\tilde{\beta}_j}\frac{F_j^2V_j^2}{H(\mathbf{V})^2}}{1-\sum_{j=1}^{m}\frac{1}{\tilde{\beta}_j}\frac{F_j^2V_j^2}{H(\mathbf{V})^2}}\right] \\
&= \left[-\frac{G(\mathbf{Q})}{1-\sum_{i=1}^{np}\frac{1}{\tilde{\alpha}_i}\frac{F_i^2Q_i^2}{G(\mathbf{Q})^2}} + \frac{H(\mathbf{V})}{1-\sum_{j=1}^{m}\frac{1}{\tilde{\beta}_j}\frac{F_j^2V_j^2}{H(\mathbf{V})^2}}\right]
\end{aligned}$$

Lemma 1: *In general case of transcendental service production function with np outputs and m inputs, the determinant of the bordered Hessian matrix of the profit* *maximization problem is given by*

$$D_{\mathcal{L}} = (-1)^{(np+m)}(\lambda)^{-(np+m+1)}T_1T_2T_3T_4T_5. \qquad (27\text{-a})$$

PROOF:
This finding results directly from equation (26-a).■

In what follows, we provide a general result on the restrictions of the coefficients of transcendental multiproduct functions that are required to satisfy the profit maximization conditions. This result is a generalization of Mundlak's (1964) result in the case of two outputs – two factor inputs.

Theorem 1: *Consider a general service generating function*

$$F(\mathbf{Q}, \mathbf{V}) = G(\mathbf{Q}) - H(\mathbf{V}) = 0 \qquad (28\text{-a})$$

where $G(\mathbf{Q}) = Q_1^{\tilde{\alpha}_1} \times \cdots \times Q_{np}^{\tilde{\alpha}_{np}} exp(\tilde{\gamma}_1 Q_1 + \cdots + \tilde{\gamma}_{np} Q_{np})$*;* $H(\mathbf{V}) = V_1^{\tilde{\beta}_1} \times \cdots \times V_m^{\tilde{\beta}_m} exp(\tilde{\omega})$*;* Q_i *is the i-th service output of the store,* $(i = \overline{1, np})$*;* V_j *is the j-th service input of the store,* $(j = \overline{1, m})$*. If the parameters satisfy the following conditions*

(a) $\tilde{\alpha}_i < 0$ *for all* $i = \overline{1, np}$*;*

(b) $\tilde{\beta}_i > 0$ *for all* $j = \overline{1, m}$*,*

then the condition for profit maximization are satisfied.

PROOF:
We consider $\lambda > 0$ and an increasing returns to scales industry, i.e., $\sum_{i=1}^{np} \tilde{\beta} j \geq 1$. We assume $\lambda > 0$ and, the first-order conditions for maximizing profit imply that $F_i > 0$ and $F_j < 0$, i.e.,

$$\left(\frac{\tilde{\alpha}_i}{Q_i} + \tilde{\gamma}_i \right) > 0, \quad i = \overline{1, np} \qquad (29\text{-a})$$

$$\frac{\tilde{\beta}_j}{V_j} > 0, \quad j = \overline{1, m}. \qquad (30\text{-a})$$

In other words, we need to have

$$\tilde{\gamma}_i > \left| \frac{\tilde{\alpha}_i}{Q_i} \right|, \quad i = \overline{1, np} \qquad (31\text{-a})$$

$$\tilde{\beta}_j > 0, \quad j = \overline{1, m}. \qquad (32\text{-a})$$

The first order condition (31-a) excludes the possibility that $\tilde{\gamma}_i = 0$ for all i.[2] This

[2] If $\tilde{\gamma}_i = 0$ for all i then $\tilde{\alpha}_i > 0$ for all i (see Proposition 1).

implies that $T_1 > 0$, $T_2 > 0$, $T_4 > 0$. The term $T_5 < 0$, because $T_3 < 0$ and $T_2 > 0$, i.e., T_5 is a sum of two negative numbers. Therefore, the sign of determinant of the bordered Hessian matrix $D_{\mathfrak{L}}$ is $(-1)^{np+m}$, which is the second order requirement for profit maximization.■

Proposition 1: *If the service function is simple Cobb-Douglas in outputs ($\tilde{\gamma}_i = 0$ for all i) and inputs and the first-order conditions are satisfied, then optimal service quantity $\mathbf{Q}^*$ is sold at the minimum cost and any inputs $\mathbf{V}^*$ yield minimum revenues. The profit $\pi(Q^*, V^*)$ at the point (Q^*, V^*) is a saddle point*

$$\pi(Q^*, V) \leq \pi(Q^*, V^*) \leq \pi(Q, V^*).$$

PROOF:
If $\tilde{\gamma}_i = 0$ for all i, then from the first-order condition (29-a) we have that $\tilde{\alpha}_i > 0$ for all i. In this case, $sign(T_2) = (-1)^{np}$, and the $sign(D_{\mathfrak{L}})$ is different from $(-1)^1$ (condition for minimum) and $(-1)^{(np+m)}$ (condition for maximum).■

A direct consequence of the Proposition 1 is that when the inputs $\mathbf{V}$ produce minimum revenues and the first-order conditions are satisfied then the profit can be maximized by a selection of products, i.e., a corner solution. This problem does not exists in the case of single product.

Proposition 2: *The condition $\tilde{\alpha}_i < 0$ and $\tilde{\gamma}_i > 0$ for all i is not the only second order condition for profit maximization.*
PROOF:
This result is also a direct consequence of Theorem 1. It is important to note that the result in Theorem 1 holds some $\tilde{\alpha}_i$ can be positive and, in this case, the corresponding $\tilde{\gamma}_i$ can be set to zero, which can be useful to reduce the number of parameters.■

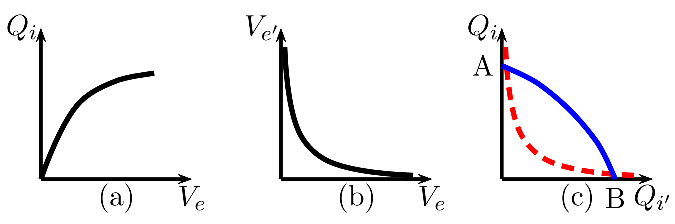

Figure A.1: Marginal rate of product (factor) substitution

Product (factor) substitution. Using the total differentiation of the service-generating function, we can obtain the marginal rate of product (factor) substitution, i.e.,

$$\begin{array}{lll} \text{Product-factor:} & \frac{dQ_i}{dV_e} = -\frac{F_e}{F_i} > 0 \\ \text{Factor-factor:} & \frac{dV_{e'}}{dV_e} = -\frac{F_e}{F_{e'}} < 0 \\ \text{Product-Product :} & \frac{dQ_i}{dQ_{e'}} = -\frac{F_{i'}}{F_i} < 0. \end{array} \tag{33-a}$$

To evaluate convexity of the different marginal rate of substitution, we compute the second derivatives, i.e.,

$$\begin{array}{lll} \text{Product-factor:} & \frac{d^2Q_i}{dV_j^2} = -\frac{F_{jj}}{F_i} \\ \text{Factor-factor:} & \frac{d^2V_{j'}}{dV_j^2} = -\frac{F_{jj}}{F_{j'}} + \frac{F_jF_{j'j}}{F_{j'}^2} \\ \text{Product-Product :} & \frac{d^2Q_i}{dQ_{i'}^2} = -\frac{F_{i'i'}}{F_i^2} + \frac{F_{ii'}}{F_{i'}}, \end{array} \tag{34-a}$$

where

$$\begin{array}{l} F_{jj} = \frac{H(\mathbf{V})}{V_j^2}\tilde{\beta}_j(1-\tilde{\beta}_j) \\ F_i = G(\mathbf{Q})\left(\frac{\tilde{\alpha}_i}{Q_i} + \tilde{\gamma}_i\right) \\ \frac{F_{jj}}{F_{j'}} = \frac{V_{j'}}{V_j}\frac{\tilde{\beta}_j}{\tilde{\beta}_{j'}}(\tilde{\beta}_j - 1) \\ \frac{F_jF_{j'j}}{F_{j'}^2} = \frac{\tilde{\beta}_j^2}{V_j^2}\frac{1}{\tilde{\beta}_{j'}} \\ -\frac{F_{i'i'}}{F_i^2} + \frac{F_{ii'}}{F_{i'}} = G(\mathbf{Q})\frac{\tilde{\alpha}_{i'}}{Q_{i'}}\frac{1}{\left(\frac{\tilde{\alpha}_i}{Q_i}+\tilde{\gamma}_i\right)}. \end{array} \tag{35-a}$$

In the case of Cobb-Douglas in inputs $0 < \tilde{\beta}_j < 1$,

$$\begin{array}{ll} \text{Product-factor:} & \frac{d^2Q_i}{dV_j^2} < 0 \\ \text{Factor-factor:} & \frac{d^2V_{j'}}{dV_j^2} > 0, \end{array} \tag{36-a}$$

which implies that product-factor rate of substitution is a concave function (Figure A.1.(a)), factor-factor rate of substitution is convex (Figure A.1.(b)). The properties of the product-product rate of substitution depends on $\tilde{\gamma}_i$, i.e.,

$$\frac{d^2Q_i}{dQ_{i'}^2} = G(\mathbf{Q})\frac{\tilde{\alpha}_{i'}}{Q_{i'}}\frac{1}{\left(\frac{\tilde{\alpha}_i}{Q_i} + \tilde{\gamma}_i\right)}. \tag{37-a}$$

If $\tilde{\gamma}_i = 0$ then from the first order condition we have $\tilde{\alpha}_i > 0$, which yields $d^2Q_i/dQ_{i'}^2 > 0$. Therefore, $\tilde{\gamma}_i = 0$ implies that product-product rate of substitution is a convex function (Figure A.1.(c)). If $\tilde{\gamma}_i > 0$ then from the first order condition we have $\tilde{\alpha}_i < 0$, which yields $d^2Q_i/dQ_{i'}^2 < 0$. In this case, product-product rate of substitution is a concave function (AB curve in Figure A.1.(c)).

Appendix B: Sales-generating function

This appendix presents the derivation of the sales-generating function using the multiproduct service technology and a demand system. The main aim is to develop a multiproduct sales function and identify its parameters. Separately identifying the coefficients of production technology and demand without price data is beyond the scope of this paper.

The multiproduct service technology is given by

$$\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i q_{ijt} + \tilde{\alpha}_y Y_{jt} = \tilde{\beta}_l l_{jt} + \tilde{\beta}_k k_{jt} + \tilde{\beta}_a a_{jt} + \tilde{\omega}_{jt} + \tilde{u}^p_{jt}, \tag{38-a}$$

where q_{ijt} is the logarithm of quantity of product category i sold by store j in period t, Y_{jt} denotes total sales of store j in period t, l_{jt} is the logarithm of the number employees, k_{jt} is the logarithm of capital stock, a_{jt} is the logarithm of the sum of the inventory level in the beginning of period t (n_{jt}) and the products bought during period t, and $\tilde{u}^p_{jt}$ are i.i.d. remaining service output shocks. Variable np_{jt} denotes the number of products (categories) of store j.[3]

We use a CES demand system to obtain an expression for the logarithm of the price of product category i (p_{ijt}), i.e., $p_{ijt} = -\frac{1}{\sigma}(q_{ijt} - q_{0t}) + \mathbf{x}'_{ijt}\frac{\tilde{\boldsymbol{\beta}}_x}{\sigma} + \frac{\sigma_a}{\sigma}a_{jt} + \frac{1}{\sigma}\tilde{\mu}_{ijt}$. Multiplying the logarithm of price by $\tilde{\alpha}_i$ and summing up over store j's product categories, we obtain the following expression:

$$\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i p_{ijt} = -\frac{1}{\sigma}\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i q_{ijt} + \frac{1}{\sigma}\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i q_{0t} + \sum_{i=1}^{np_{jt}} \tilde{\alpha}_i \mathbf{x}'_{ijt}\frac{\tilde{\boldsymbol{\beta}}_x}{\sigma} + \frac{\sigma_a}{\sigma}\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i a_{jt} + \frac{1}{\sigma}\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i \tilde{\mu}_{ijt}. \tag{39-a}$$

The logarithm of sales per product category is $y_{ijt} = q_{ijt} + p_{ijt}$. To obtain an expression for sales per product category, we sum up the expressions (38-a) and (39-a), i.e., $\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i y_{ijt} + \left(1-\frac{1}{\sigma}\right)\tilde{\alpha}_y Y_{jt}] = \left(1-\frac{1}{\sigma}\right)[\tilde{\beta}_l l_{jt} + \tilde{\beta}_k k_{jt} + \tilde{\beta}_a a_{jt}] + \frac{1}{\sigma}\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i q_{0t}] + \sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i \mathbf{x}'_{ijt}\frac{\tilde{\boldsymbol{\beta}}_x}{\sigma}] + \frac{\sigma_a}{\sigma}\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i]a_{jt} + \frac{1}{\sigma}\sum_{i=1}^{np_{jt}}\tilde{\alpha}_i\tilde{\mu}_{ijt} + (1-\frac{1}{\sigma})\tilde{\omega}_{jt} + (1-\frac{1}{\sigma})\tilde{u}^p_{jt}$. The logarithm of the aggregate quantity of the outside option q_{0t} can be written as $q_{0t} = \tilde{c}_{ij}q_{i0t}$, where $\tilde{c}_{ij} > 1$ and q_{i0t} is the logarithm of the quantity of product category i that is sold by stores in the outside option.[4] Thus, we can write $\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i q_{0t}] = \sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i\tilde{c}_{ij}q_{i0t}]$. Using $q_{i0t} = y_{i0t} - p_{i0t}$, we obtain $\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i q_{0t}] = \sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i\tilde{c}_{ij}(y_{i0t} - p_{i0t})]$. Because $\tilde{c}_{ij} > 1$, there exist $s_{ij} < 1$ and $c_j > 1$ such that $\sum_{i=1}^{np_{jt}}\tilde{\alpha}_i\tilde{c}_{ij} = c_j$ and $\sum_{i=1}^{np_{jt}} s_{ij} = 1$. Therefore, we obtain $\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i q_{0t}] = c_j(\sum_{i=1}^{np_{jt}} s_{ij}y_{i0t} - \sum_{i=1}^{np_{jt}} s_{ij}p_{i0t}) = c_j(\tilde{y}_{0jt} - \tilde{p}_{0jt}) = c_j y_{0jt} \equiv y_{ot}$, where $\tilde{y}_{0jt}$ are weighted sales of product categories of store j that are sold in the outside

[3] As we mention in the main text, we have only information on product categories in the empirical application.

[4] Note that store j does sell only few product categories, and therefore $c_{ij} > 1$.

option, $\tilde{p}_{0jt}$ is a weighted price index, y_{0jt} are deflated sales of product categories of store j that are sold in the outside option, and y_{ot} denotes outside option sales. We measure y_{ot} by total sales of stores in the outside option. Most importantly, for any store j, we can write the term of the outside option in terms of total sales of the outside option in the multiproduct sales function. If there are no stores in the outside option, y_{ot} represents total sales in the market.

The next step is to regroup the remaining coefficients and determine how they are affected by σ. We denote $\beta_q \equiv 1/\sigma$, $\beta_l \equiv (1-\frac{1}{\sigma})\tilde{\beta}_l$ and $\beta_k \equiv (1-\frac{1}{\sigma})\tilde{\beta}_k$. As we mention in the main text, we are unable to identify the impact of inventory separately on demand and supply without additional assumptions. Therefore, we sum up the net impact of inventory on sales under parameter $\overline{\beta}_a$, i.e., we denote $(1-\frac{1}{\sigma})\overline{\beta}_a \equiv (1-\frac{1}{\sigma})\tilde{\beta}_a + \frac{\sigma_a}{\sigma}\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i$. Furthermore, to shorten the notation, we denote $\beta_a \equiv (1-\frac{1}{\sigma})\overline{\beta}_a$. Because a_{jt} is part of both supply and demand equations, we are unable to identify $\tilde{\beta}_a$ and σ_a separately. In other words, we can identify only the net effect $\overline{\beta}_a$. In our case, $\mathbf{x}_{ijt}$ includes only market variables and therefore we denote $\overline{\boldsymbol{\beta}}_x \equiv \sum_{i=1}^{np_{jt}} \tilde{\alpha}_i \tilde{\boldsymbol{\beta}}_x$ and $\boldsymbol{\beta}_x \equiv \overline{\boldsymbol{\beta}}_x/\sigma$. We also denote by $\omega_{jt} \equiv (1-1/\sigma)\tilde{\omega}_{jt}$ a measure of revenue (sales) productivity, and refer to it as simple store productivity in what follows. Additionally, μ_{jt} is a weighted sum of all unobserved product demand shocks at the store level, determined as $\mu_{jt} \equiv (1/\sigma)\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i \mu_{ijt}$, and measures store j's specific demand shocks in period t, and u^p_{ijt} are i.i.d. remaining shocks to sales that are mean-independent of all control variables and store inputs. Using this notation we can write the multiproduct sales function as $\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i y_{ijt} + \left(1-\frac{1}{\sigma}\right)\tilde{\alpha}_y Y_{jt}] = \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \beta_q y_{ot} + \mathbf{x}'_{jt}\boldsymbol{\beta}_x + \omega_{jt} + \mu_{jt} + u^p_{ijt}$.

The combination of the service technology and a simple CES demand yields an expression for the sales technology where the left-hand-side is a linear combination of sales per product category and the right-hand-side is a linear combination of store inputs, local demand shifters, store revenue productivity, and demand shocks. This relationship solves the aggregation problem across different products. How many output parameters $\tilde{\alpha}_i$ we can identify depends on the available data on products (categories) and the variation across stores. If there is large heterogeneity in products offered for sale across stores, we need to reduce the number of parameters $\tilde{\alpha}_i$ that can be identified. By choosing only stores that sell similar products, we induce a selection problem. As a result, even if we estimate many technology parameters, the overall inference of the empirical exercise might be biased. In our Swedish data, there is large heterogeneity in product categories stores offer for sale. Thus, since we solve the multiproduct aggregation problem across product categories using sales instead of quantity, we rewrite the linear expression for product sales to reduce the number of parameters. In other words, we focus on sales of product category i and sales of other product categories. To obtain an estimable product sales equation that includes the logarithm of sales of product category i, y_{ijt}, and the logarithm of sales of other product categories inside the store y_{-ijt}, we rewrite the linear sum of product category sales $\sum_{i=1}^{np_{jt}} \left[\tilde{\alpha}_i y_{ijt} + \left(1 - \frac{1}{\sigma}\right) \tilde{\alpha}_y Y_{ijt}\right] \equiv \alpha_i y_{ijt} + \alpha_y y_{-ijt}$. Using new transformations, we can rewrite the sales of product category i as[5]

$$y_{ijt} = -\alpha_y y_{-ijt} + \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \beta_q y_{ot} + \mathbf{x}'_{jt}\boldsymbol{\beta}_x + \omega_{jt} + \mu_{jt} + u^p_{ijt}, \qquad \text{(40-a)}$$

which is the equation we estimate in the main text.

In summary, it is important to discuss a few aspects of identification of the multi-product technology. First, we focus on developing a simple multiproduct setting that does not require detailed product data and that can be used to analyze trends and the impact of policies in local markets. Second, we need product prices to identify the initial quantity weights $\tilde{\alpha}_i$ and variation in other product characteristics. Most importantly, in empirical settings, even if we have access to detailed product data and prices, we need data over a long period of time to consistently identify $\tilde{\alpha}_i$ (solving a system of equations at the firm/store level). In our setting, the scope parameter α_y in the multiproduct sales-generating function (40-a) includes the sum of weights $\tilde{\alpha}_i$. In other words, parameter α_y provides information on the economies of scope in the store based on supply-side information (the multiproduct service frontier) and demand (elasticity of substitution).

B.1: Monte Carlo simulation

Table B.1: Estimation of single and multiproduct production function using two-step estimator (source: Maican and Orth (2019))

	DGP: Single-product		DGP: Multi-product	
	Estim.	Std.	Estim.	Std.
Log of labor (β_l)	0.599	0.008	0.601	0.026
Log of capital (β_k)	0.400	0.005	0.401	0.031
Log of other products (α_y)			-0.854	0.078

NOTE: The two-step estimator uses non-parametric labor demand function to proxy for productivity (Doraszelski and Jaumandreu, 2013; Maican and Orth, 2015; Maican and Orth, 2017). Reported standard errors are computed based on 1000 simulations. Monte Carlo simulations use $\beta_l = 0.6$, $\beta_k = 0.4$, $\alpha_y = -0.85$. Single-product function is estimated at the firm level. Multi-product function is estimated at the product level assuming the same production technology across products. The number of firms is 1000. For the multiproduct DGP, the number of products for each firm is 3. Labor is simulated using first-order condition profit maximization. Investment is simulated based on policy function that is increasing the in firms state variables. Capital stock is constructed using perpetual inventory method $K_{jt} = (1 - 0.2)K_{jt-1} + I_{jt-1}$. Productivity follows an AR(1) process with the persistence $\rho = 0.7$ and it is simulated to have constant variance over time (standard deviation 0.3). Wages follow an AR(1) process with the persistence $\rho = 0.3$ and it is simulated to have constant variance over time (standard deviation 0.3). The number of years is 10 (all variables are used in steady state).

[5]We normalize $\alpha_i = 1$.

Table B.2: Estimation of single output production when DGP is a multiproduct production function (source: Maican and Orth (2019))

	Estim.	Std.
Log of labor (β_l)	0.490	0.004
Log of capital (β_k)	0.542	0.002
Distribution of productivity bias		
Q_{25}	Q_{50}	Q_{75}
-0.085	0.116	0.322

NOTE: The two-step estimator uses non-parametric labor demand function to proxy for productivity (Doraszelski and Jaumandreu, 2013; Maican and Orth, 2015); Maican and Orth, 2017). Reported standard errors are computed based on 1000 simulations. Monte Carlo simulations use $\beta_l = 0.6$, $\beta_k = 0.4$, $\alpha_y = -0.85$. Single-product function is estimated at the firm level. Multi-product function is estimated at the product level assuming the same production technology across products. The number of firms is 1000. For the multiproduct DGP, the number of products for each firm is 3. Labor is simulated using first-order condition profit maximization. Investment is simulated based on policy function that is increasing the in firms' state variables. Capital stock is constructed using perpetual inventory method $K_{jt} = (1 - 0.2)K_{jt-1} + I_{jt-1}$. Productivity follows an AR(1) process with the persistence $\rho = 0.7$ and it is simulated to have constant variance over time (standard deviation 0.3). Wages follow an AR(1) process with the persistence $\rho = 0.3$ and it is simulated to have constant variance over time (standard deviation 0.3). The number of years is 10 (all variables are used in steady state).

Appendix C: Invertibility conditions with two unobservables

The general labor demand and inventory functions that arise from the stores' dynamic optimization problem are

$$
\begin{aligned}
l_{jt} &= \tilde{l}_t(\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt}) \\
a_{jt} &= \tilde{a}_t(\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt}).
\end{aligned}
\tag{41-a}
$$

The main aim is to recover ω_{jt} and μ_{jt} using this system of equations. The conditions required for identification can be grouped as follows: (i) general conditions that the policy functions of the dynamic programming problem have to satisfy; (ii) conditions that the system of equations should satisfy to have a unique solution. In what follows, we discuss these conditions.

First, to back out ω_{jt} and μ_{jt}, the assumption 6 must hold, i.e., the policy functions $\tilde{l}_t(\cdot)$ and $\tilde{a}_t(\cdot)$ must be strictly monotonic in ω_{jt} and μ_{jt}, which holds under mild regularity conditions on the dynamic programming problem (Pakes, 1994; Maican, 2016). The static profits are assumed to be strictly increasing in ω_{jt}, μ_{jt}, and k_{jt} and continuously differentiable in these variables. Another condition is supermodularity of the static profits with respect to ω_{jt} and μ_{jt}, i.e., the impact of productivity on profits is increasing in μ_{jt}. In other words, stores with large demand shocks experience a larger increase in profits due to productivity. This assumption is not restrictive since stores that experience large demand shocks invest to increase their productivity to satisfy the demand. Furthermore, static profits are assumed to be supermodular with respect to ω_{jt} (μ_{jt}) and k_{jt}, i.e., marginal product of capital is increasing in productivity and demand shocks. This condition can also be interpreted as, stores with larger capital stock have higher profits due to an increase in productivity or demand shocks. All these conditions (strict monotonicity and supermodularity) on static profits yield that value and policy functions are strictly increasing in ω_{jt}, μ_{jt}, and k_{jt} (Pakes, 1994; Maican, 2016).

Second, we discuss the general properties that must be satisfied by labor demand ($\tilde{l}(\cdot)$) and inventory ($\tilde{a}(\cdot)$) functions such that the system (41-a) has a unique solution. This system can be solved for ω_{jt} and μ_{jt} in terms of k_{jt}, n_{jt}, l_{jt}, w_{jt}, and a_{jt} when certain partial derivatives are continuous and the 2×2 Jacobian determinant $\partial(\tilde{l},\tilde{a})/\partial(\omega,\mu)$ is not zero. In other words, the ratios between the impact of ω and μ on the investment and inventories should not be the same, i.e., $(\partial\tilde{l}/\partial\omega)/(\partial\tilde{l}/\partial\mu) \neq (\partial\tilde{a}/\partial\omega)/(\partial\tilde{a}/\partial\mu)$. Therefore, this condition requires that productivity and demand shocks have a different impact on investment and inventory, and the relative impact is not the same.

We apply implicit function theorem to prove the invertibility of the system (41-a). In our case, points in (2+5)-dimensional space $\mathbb{R}^{2+5}$ can be written in the form of $(\mathbf{x};\mathbf{b})$, where $\mathbf{x} = (\omega,\mu)$ and $\mathbf{b} = (k,n,l,a,w)$. We can rewrite the system as $f_1(\mathbf{x};\mathbf{b}) = 0$ and $f_2(\mathbf{x};\mathbf{b}) = 0$ or simply as an equation $F(\mathbf{x};\mathbf{b}2) = 0$. To understand the invertibility of the policy functions, we need to know when the relation $F(\mathbf{x};\mathbf{b}) = 0$ is a also a function. In other words, what are the conditions such that $F(\mathbf{x};\mathbf{b}) = 0$ can be solved explicitly for $\mathbf{b}$ in terms of $\mathbf{x}$ obtaining a unique solution. The Theorem C.1 (implicit function theorem) provides the conditions that for a given point $(\mathbf{x}_0,\mathbf{b}_0)$ such that $F(\mathbf{x}_0,\mathbf{b}_0) = 0$ there exists a neighborhood of $(\mathbf{x}_0,\mathbf{b}_0)$ where the relation $F(\mathbf{x};\mathbf{b}) = 0$ is a function.

Theorem C.1. *Let $\mathbf{f} = (f_1, f_2)$ be a vector of functions defined on the open set S in $\mathbb{R}^{2+5}$ with values in $\mathbb{R}^2$. Suppose $\mathbf{f} \in C'$ on S. Let $(\mathbf{x}_0;\mathbf{b}_0)$ be a point in S for which $\mathbf{f}(\mathbf{x}_0,\mathbf{b}_0) = \mathbf{0}$ and for which the 2×2 Jacobian determinant $\partial(f_1,f_2)/\partial(\omega,\mu)$ is not zero at $(\mathbf{x}_0,\mathbf{b}_0)$. Then there exists a 5-dimensional open set B_0 that includes $\mathbf{b}_0$ and one and only one vector based functions $\mathbf{g}$ defined on B_0 and having values in $\mathbb{R}^2$ such that*
(i) $\mathbf{g} \in C'$ *on* B_0
(ii) $\mathbf{g}(\mathbf{b}_0) = \mathbf{x}_0$
(iii) $\mathbf{f}(\mathbf{g}(\mathbf{b});\mathbf{b}) = \mathbf{0}$ *for every* $\mathbf{b}$ *in* B_0.
PROOF: This theorem is, in fact, the implicit function theorem applied on our case. The general proof of the theorem can found in Apostol (1974).

Appendix D: Entry regulation: Plan and Building Act (PBL)

The majority of OECD countries have entry regulations that empower local authorities to decide on store entry. However, the regulations differ substantially across countries (Boylaud and Nicoletti, 2001; Griffith and Harmgart, 2005; Schivardi and Viviano, 2011). While some countries strictly regulate large entrants, more flexible zoning laws exist, for instance, in the U.S. (Pilat, 1997).

The Swedish Plan and Building Act (PBL) regulates the use of land and water and buildings. The PBL consists of the planning requirements for land and water areas as well as buildings. The ultimate goal of PBL is to promote equal and adequate living conditions and a lasting sustainable environment for today and future generations. The regulation contains two documents/plans: (i) the comprehensive plan and (ii) the detailed development plan. Municipalities are required to have a comprehensive plan that covers the entire municipality and that guides decisions regarding the use of land, water areas and the built environment. The comprehensive plan records public interests and national interests. Municipalities also have to provide detailed development plans that cover only a fraction of the municipality. Municipalities are divided into smaller areas. These plans indicate and set limits on the use and design of public spaces, land and water areas.

The purpose of the comprehensive plan is to provide an attractive public environment that is sustainable. It is the basis for decisions regarding the use of land, water and the development and preservation of buildings. It reflects the public interest and addresses important environmental and risk factors that must be taken into account in the planning of any endeavor. Necessary features include the housing needs of the municipal inhabitants, the protection of valuable natural and cultural environments, and providing inhabitants with access to services.

The detailed development plan consists of a map with text that indicates what, where and how one is allowed to build, as well as appropriate uses for the area. For instance, it indicates the appropriate design and use of housing, nature and water areas. Other examples include construction rights for real estate including the size and form of structures, the possibility of opening a restaurant, work places and businesses, housing, hotels, housing (villa or apartments), pre-schools, elementary schools, health care, energy- and water services, parks, streets, squares, etc.

The detailed development plan indicates whether retail stores are allowed. The right to open and operate a retail food store is addressed in the detailed development plan. Each store seeking to enter the market is required to file a formal application with the local government. For the entry to occur, the municipality can accept a new detailed development plan or make changes in an existing one. First, in the application, the store must state the purpose of the activity: retail, housing, offices, manufacturing, or other. Second, the store must describe the main purpose of its activity and what it is to contain, e.g., a new building of a certain size, wholesale provision with trucks, parking places and is obligated to be as detailed as possible. Before the new detailed development plan is approved, it must be made publicly available. Inhabitants of the municipality are allowed to express their opinions and views on the proposed changes. If some do not agree with the proposed plan, they can appeal. The municipality must then perform a new evaluation and look for alternative solutions to the question at hand.

When a retail store seeks to enter a local market, the municipality evaluates the consequences for exit, prices, local employment, availability of store types and product assortments for different types of consumers, purchasing patterns and purchasing trips, consumer travel behavior, traffic (e.g., generated traffic per square meter of the new sales space) including the effect it has on noise and air pollution for nearby consumers, as well as the number of individuals who will be affected - probable health effects, risk evaluations, broader environmental issues, increased distance to the store, parking, water, energy supply, etc.

In addition, the municipal council must evaluate the positive and negative consequences of the new entrant for different inhabitants, the environment, traffic, public transport, safety, etc. The municipality must consider whether new bus lines are necessary, as well as walking and biking paths. This is to ensure each consumer in the municipality has access to different types of stores, a broad product assortment and reasonable prices. A store entrant is prohibited from hindering real estate developments that will be useful for the public interest, i.e., housing, places of work, traffic infrastructure and leisure environments. The municipal council evaluates and gives an overall assessment of the trade-offs between the public interest and private retail interests. This assessment is based on contingency analysis, an investigation of alternative solutions and developments, and strategic judgement. It is important to evaluate the effects that accepting a new detailed development plan and changing an existing one on the public interest.

All stores are regulated by the PBL in Sweden, in contrast with, for example, the U.K., which explicitly focuses on regulating large stores (Maican and Orth, 2015; Sadun, 2015). The PBL is considered one of the major barriers to entry and is the cause of a diverse array of outcomes, e.g., price levels, across municipalities (Swedish Competition Authority, 2001:4). Several reports stress the need to better analyze how entry regulation affects market outcomes (Pilat, 1997; Swedish Competition Authority, 2001:4; Swedish Competition Authority, 2004:2).

Appendix E: Additional policy experiments: Cost subsidies

The last two counterfactual experiments introduce a subsidy to the marginal adjustment cost of product categories. In CF_5 all stores receive the same subsidy level per product category, i.e., we set the intercept equal to zero in equation (17). The results in Table E.3 (Panel A) show that approximately 16 percent of the stores adjust their number of product categories as a result of the subsidy. Product repositioning occurs both in terms of entry and exit of product categories. On average, the entry and exit rates are approximately the same. There are also changes in the intensive product-category margin. Sales per product category and incumbents' long-run profits increase by 3-5 percent. Profitability gains are driven by changes in the intensive and extensive product-category margins and lower adjustment costs. The subsidy reduces adjustment costs by 13 percent in rural markets and by almost 10 percent in restrictive markets, whereas it only decreases by half as much for the other market types.

Policy experiment CF_6 incentivizes large stores to adjust their product categories, i.e., the subsidy level depends on the number of products offered. We subsidize the part of marginal cost that is increasing in store variety by a 35 percent reduction of the coefficient φ_2 in equation (17). The CF_6 is equivalent to the first counterfactual CF_5 in terms of aggregate cost savings, which is related to governmental cost, approximately SEK 7 million on average per year. Table E.3 (Panel B) shows higher product-category entry rates than exit rates in CF_6 than in CF_5. Product-category entry rates are higher in rural and restrictive markets (approximately 3.5 percent) than in urban and liberal markets (approximately 2.5 percent). Product-category repositioning increases sales per product category by between 2.4 and 6.5 percent, on average. Incumbents offering large variety benefit from the subsidy, especially in rural and restrictive markets. Profitability gains are highest in rural markets with an average increase in store value of 7.6 percent. Incumbents' long-run profits increase by 1 percent in urban and restrictive markets and by 3.6 percent in liberal markets. A subsidy design that targets the size of variety as in CF_6, rather than the same subsidy per product category, particularly improves variety in rural and restrictive markets. The reason is that large incumbents better utilize benefits from economies of scope to increase variety.

A comparison of the findings regarding the cost subsidy designs shows that the magnitudes of the induced changes are lower when implementing CF_5 and CF_6 than when implementing CF_4 (discussed in the main text). For instance, the share of stores that adjust their product categories is smaller and the profitability gains are lower in CF_5 and CF_6 than in CF_4.

Table E.3: Counterfactual experiments: Cost subsidies for product variety

	Type of market							
	Rural		Urban		Restrictive		Liberal	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std
Panel A: CF_5 - Same subsidy per product category								
Share of stores with product adjust.	0.1627		0.1607		0.1758		0.1462	
Product category entry rate	0.0193	0.0788	0.0270	0.1216	0.0292	0.1346	0.0220	0.0918
Product category exit rate	0.0213	0.0754	0.0242	0.0872	0.0270	0.0894	0.0203	0.0807
Inventory before sales	0.0041	0.1039	0.0112	0.1464	0.0087	0.1572	0.0112	0.1201
Sales	-0.0037	0.0751	0.0192	0.2879	0.0055	0.1730	0.0249	0.3297
Sales per product	0.0095	0.1351	0.0610	0.7237	0.0450	0.6006	0.0589	0.7144
Adjustment cost	-0.1338	0.1961	-0.0681	0.6483	-0.0969	0.2463	-0.0623	0.8042
Value function	0.0517	0.2118	0.0432	0.3177	0.0344	0.1899	0.0550	0.3821
Panel B: CF_6 - Subsidy per product category that varies with the number of product categories (CF_5 cost equivalent)								
Share of stores with product adjust.	0.1617		0.1538		0.1674		0.1432	
Product category entry rate	0.0331	0.1233	0.0291	0.1265	0.0362	0.1503	0.0235	0.0954
Product category exit rate	0.0163	0.0801	0.0192	0.0787	0.0183	0.0747	0.0191	0.0830
Inventory before sales	0.0111	0.1530	0.0027	0.1225	0.0035	0.1324	0.0048	0.1244
Sales	0.0129	0.1080	0.0100	0.1733	0.0053	0.1436	0.0158	0.1813
Sales per product	0.0654	0.9125	0.0353	0.4947	0.0243	0.4975	0.0568	0.6691
Adjustment cost	-0.1081	0.1704	-0.0813	0.5235	-0.0905	0.1958	-0.0817	0.6500
Value function	0.0761	0.3213	0.0133	0.3521	0.0123	0.2377	0.0364	0.4297

NOTE: Figures represent growth changes. All stores receive subsidies. All subsidy counterfactuals are based on the marginal adjustment cost in product categories. The counterfactuals CF_5 and CF_6 are cost equivalent at the industry level. In CF_5, the subsidy per product category is equal to φ_1, i.e., we set $\varphi_1 = 0$. In CF_6, the subsidy per product category is 35 percent of $\varphi_2 np_{jt}$).